



Searching for Stochastic Gravitational-Wave Background with LIGO

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for the LIGO Scientific Collaboration

Fermilab, 08/06/12



Outline

- Gravitational Waves
 - » Sources
 - » Detectors
- Searches for stochastic background of gravitational waves using LIGO data
- Outlook for the future:
 - » Advanced LIGO



Gravitational Waves

- Newtonian gravity: instantaneous action at a distance.
- General Relativity: the “signal” travels at the speed of light.
- Weak field limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Einstein’s field equations reduce to the wave equation:

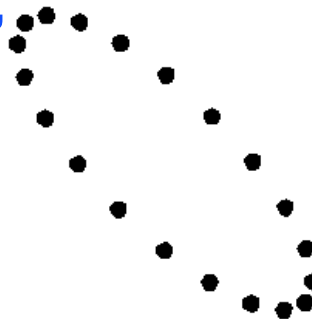
$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0$$

- Two polarizations: $h = ah_+ + bh_\times$ $a, b \sim f(\omega t - \mathbf{k} \cdot \mathbf{x})$

“+”



“X”



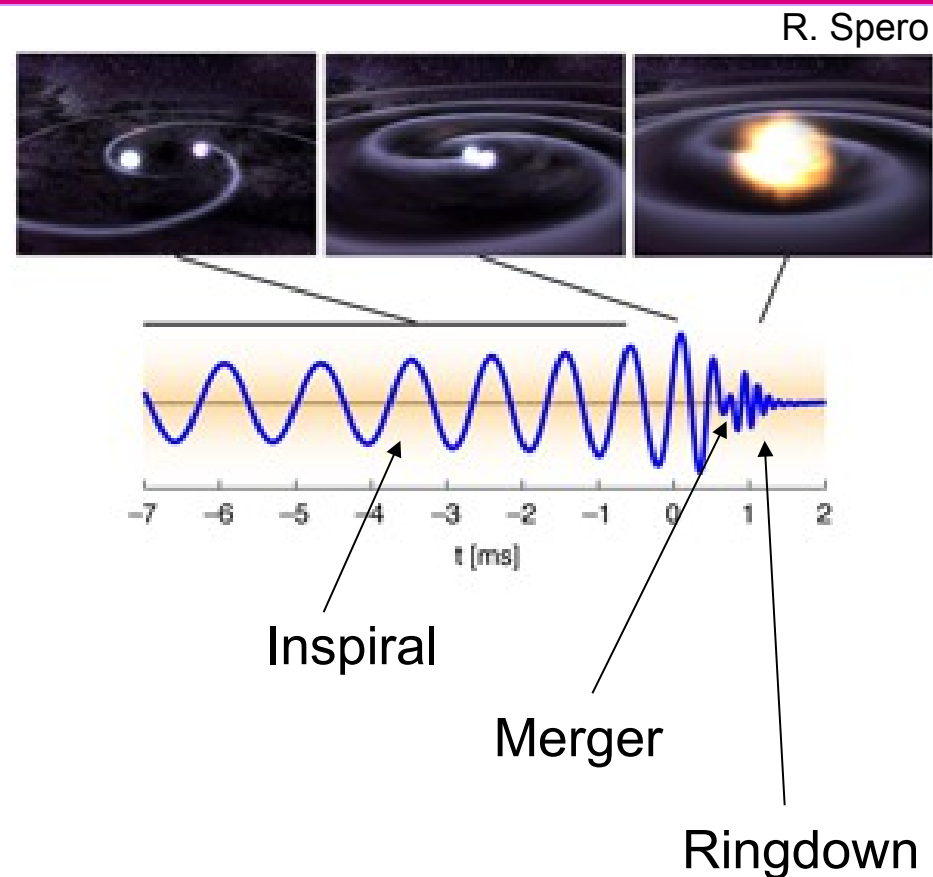


Sources of Gravitational Waves

- Transient sources, typically 1-sec long (or less):
 - » Compact binary coalescences
 - » Bursts: transient emissions during Supernovae, GRBs...
- Continuous sources:
 - » Periodic sources: pulsars
 - » Stochastic sources: cosmological and astrophysical
- New search: long-lasting transients.
 - » Time-scale of minutes, hours, or days
- Unexpected?

Compact Binary Coalescences

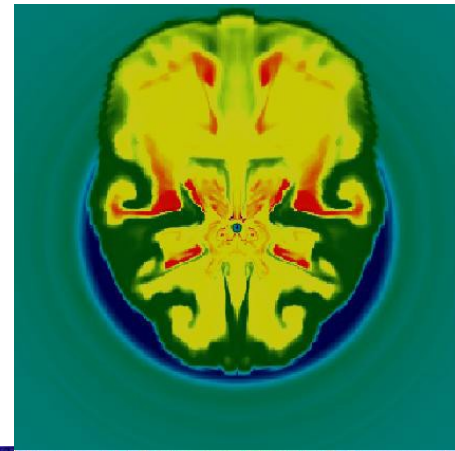
- Compact binary objects:
 - » Two neutron stars and/or black holes.
- Inspiral toward each other.
 - » Emit gravitational waves as they inspiral.
- Amplitude and frequency of the waves increases over time, until the merger.
- Waveform relatively well understood, matched template searches.
- Science:
 - » Strong field GR (BH-BH mergers).
 - » Equation of state in NS.
 - » Standard “sirens” - probe cosmology.



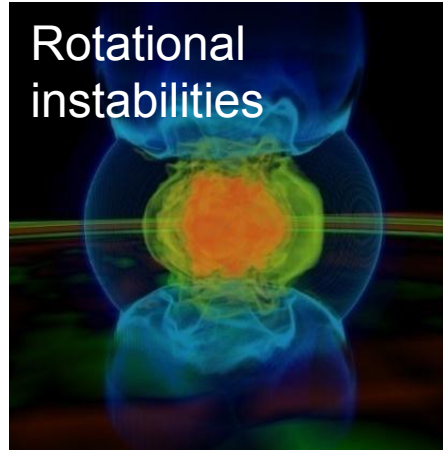
Bursts

- Many potential transient sources:
 - » Supernovae: probe the explosion mechanisms.
 - » Gamma Ray Bursts: collapse of rapidly rotating massive stars or neutron star mergers.
 - » Pulsar glitches: accretion.
 - » Cosmic strings cusps.
- Models are ok, but not essential:
 - » Search for power excess in the data.
 - » Search for any short signal with measurable strain signal.

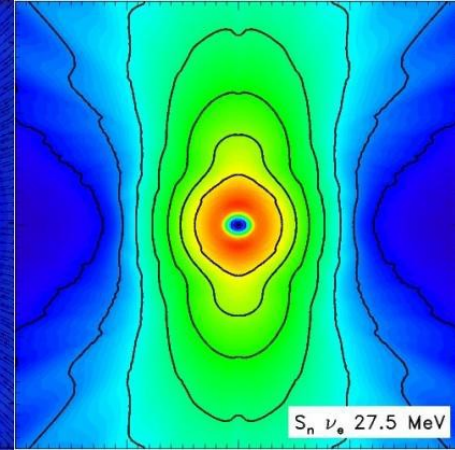
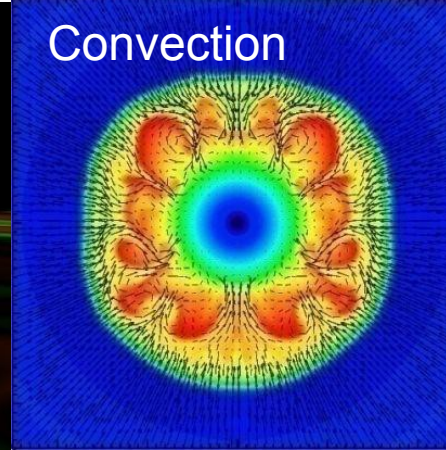
Aspherical outflows



Rotational instabilities



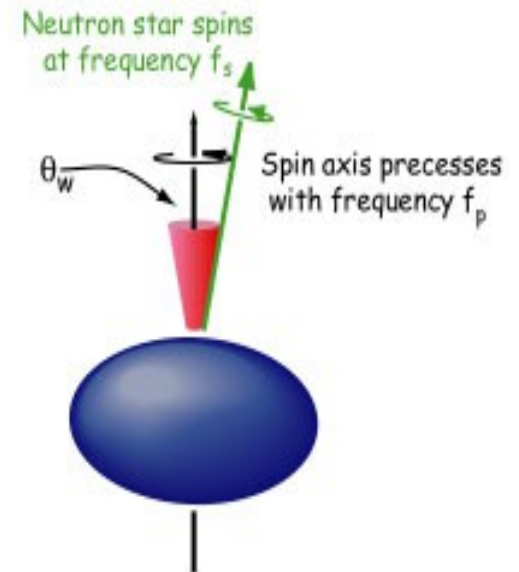
Convection



C. Ott

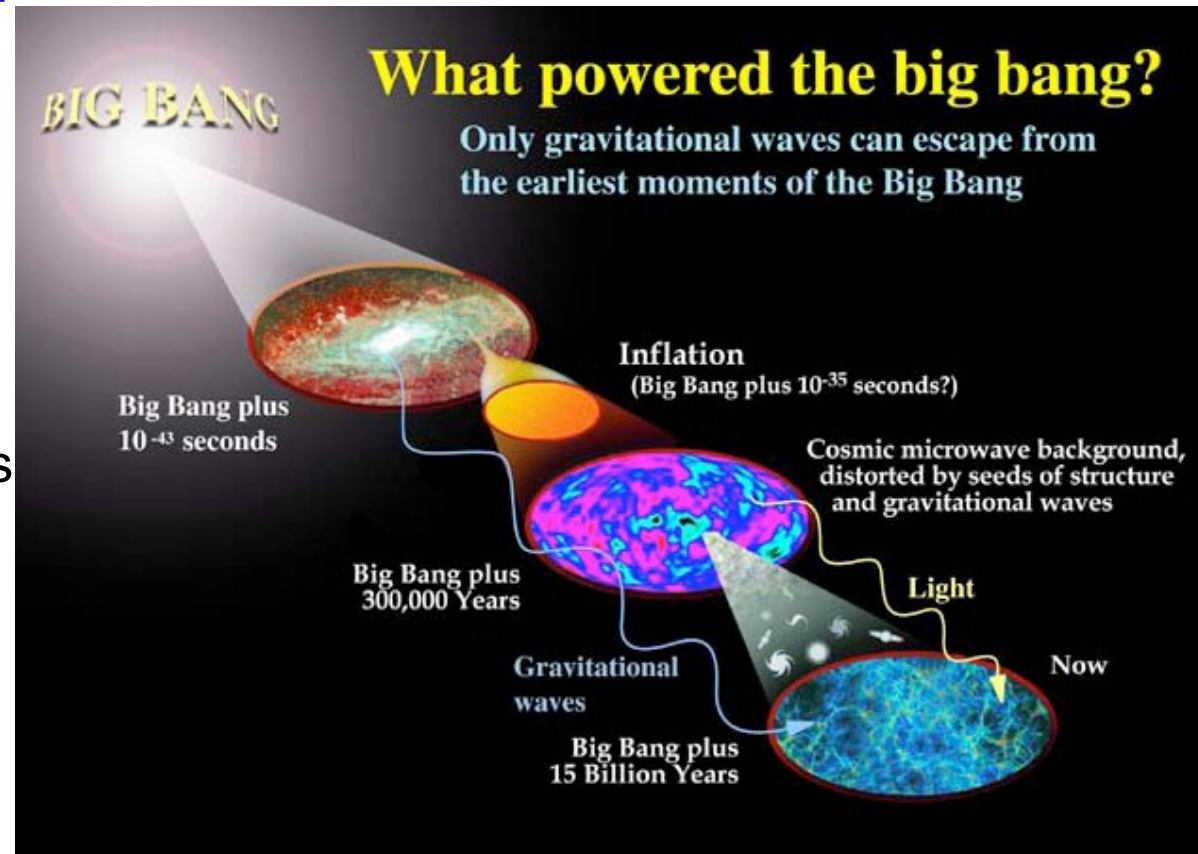
Sources: Periodic

- Pulsars with mass non-uniformity:
 - » Small “mountain”.
 - » Density non-uniformity.
 - » Dynamic processes inside neutron star, leading to various instabilities.
- Produce gravitational-waves at twice the rotational frequency.
- Waveform well understood:
 - » Sinusoidal, but Doppler-modulated.
- Continuous source!



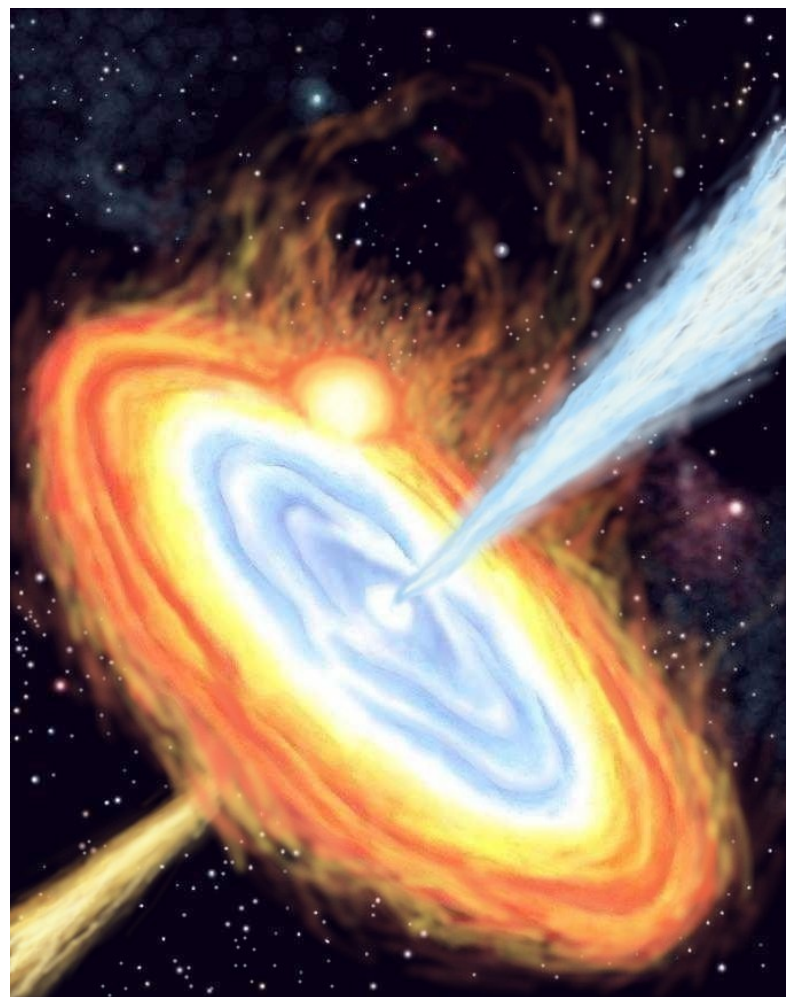
Sources: Stochastic Background

- Incoherent superposition of many unresolved sources.
- Cosmological:
 - » Inflationary epoch, preheating, reheating
 - » Phase transitions
 - » Cosmic strings
 - » Alternative cosmologies
- Astrophysical:
 - » Supernovae
 - » Magnetars
 - » Double neutron stars
- Potentially could probe physics of the very-early Universe.



Sources: Long Transients

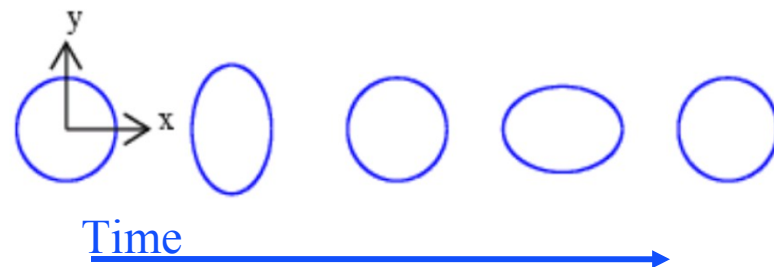
- New category of sources, received much attention recently.
 - » Tens of seconds or longer.
- Long GRBs (Piro & Pfahl):
 - » In-falling material circularizes and falls into the black hole via an accretion disk.
 - » Strong cooling from helium photodisintegration leads to disk fragmentation, and GWs.
- Magnetars:
 - » ~10% of neutron stars, strong magnetic fields ($10^{14} - 10^{16}$ G).
 - » B-field/accretion can induce tri-axial deformation – GWs.





Interferometers as Gravitational Wave Detectors

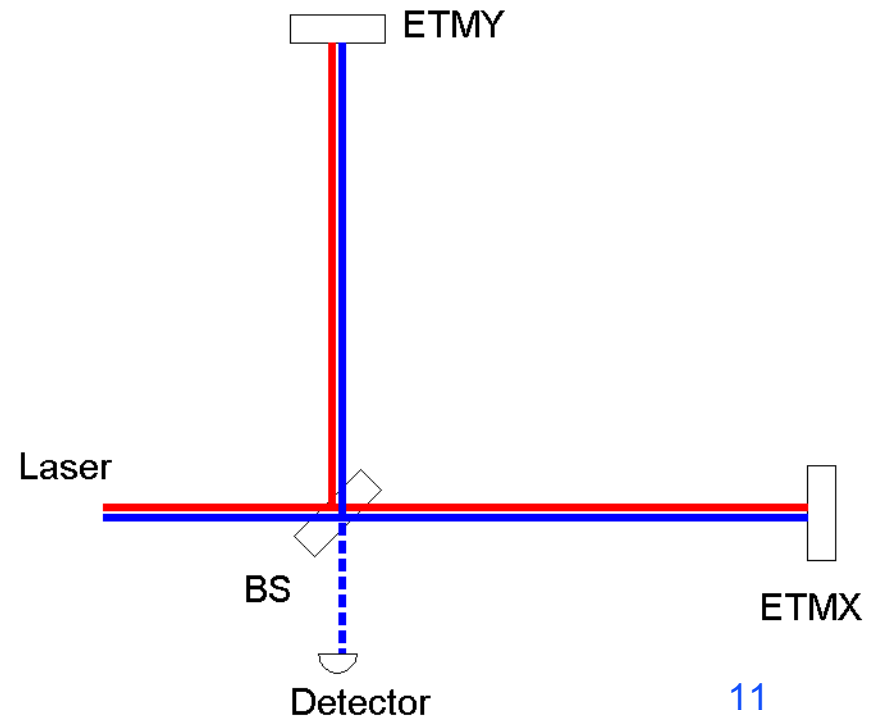
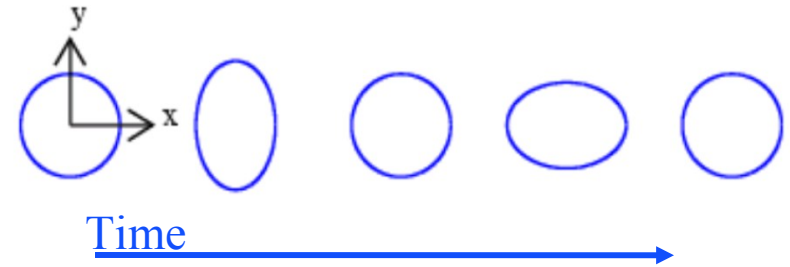
- Gravitational wave effectively stretches one arm while compressing the other.





Interferometers as Gravitational Wave Detectors

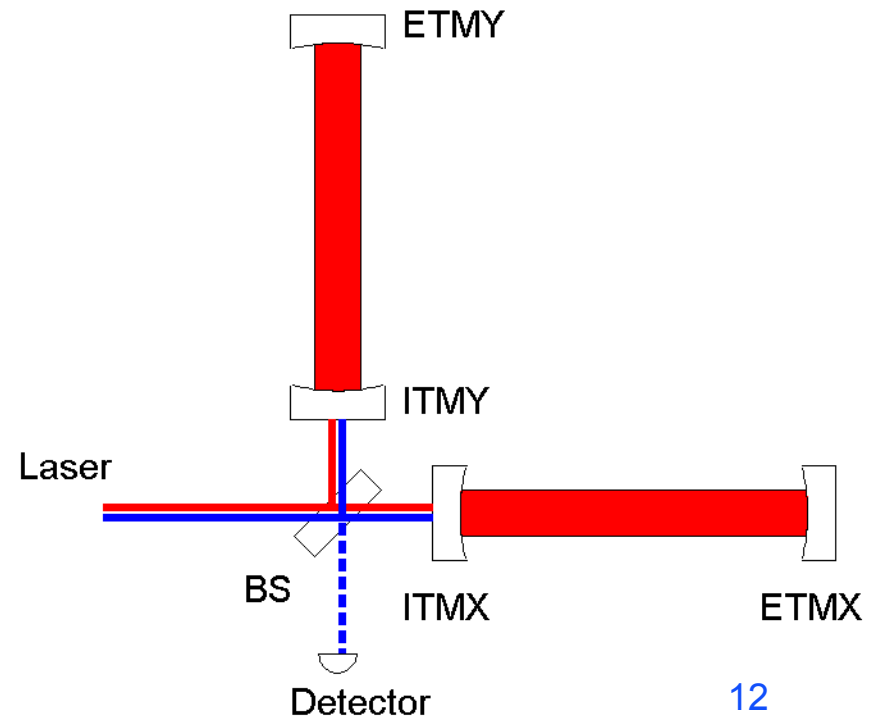
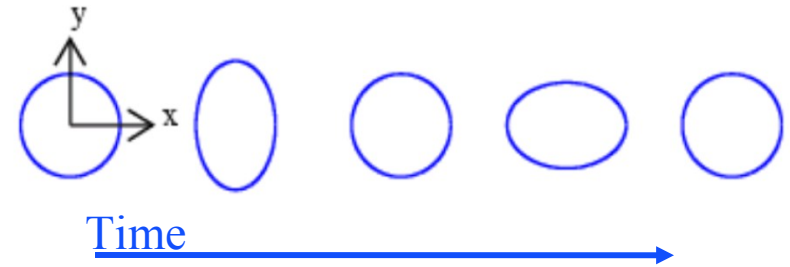
- Gravitational wave effectively stretches one arm while compressing the other.
- Interferometer measures the arm-length difference.
 - » Suspended mirrors act as “freely-falling”.
 - » Dark fringe at the detector.





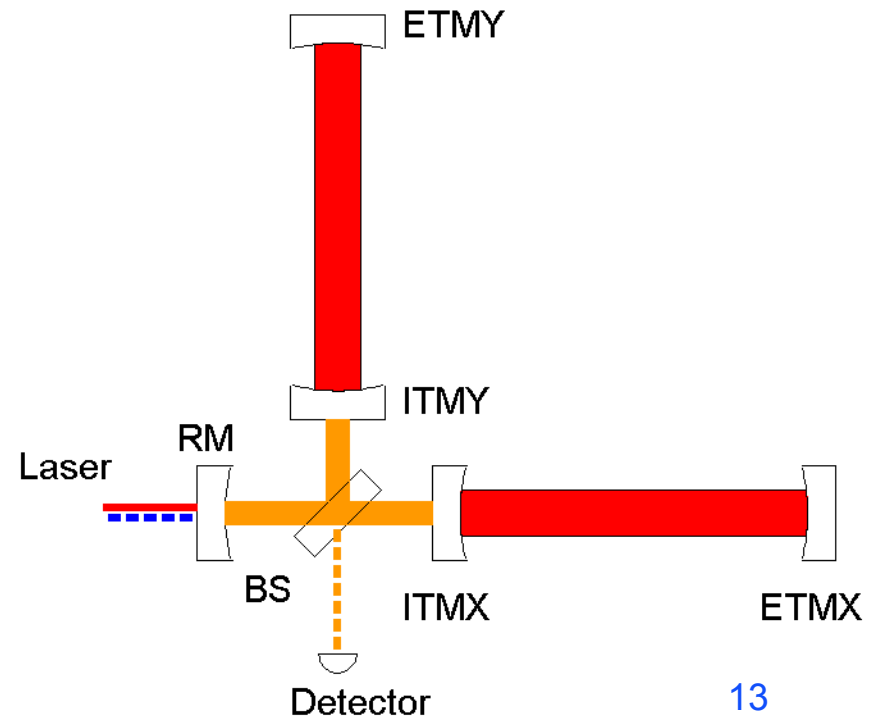
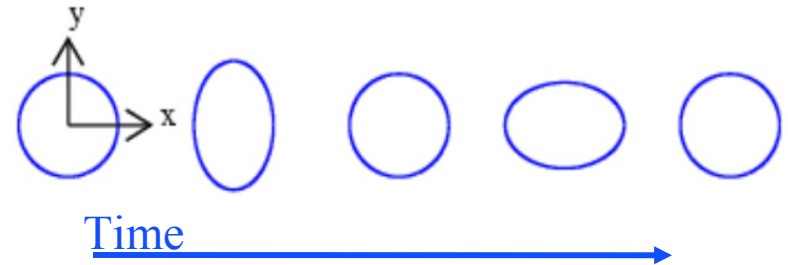
Interferometers as Gravitational Wave Detectors

- Gravitational wave effectively stretches one arm while compressing the other.
- Interferometer measures the arm-length difference.
 - » Suspended mirrors act as “freely-falling”.
 - » Dark fringe at the detector.
- Fabry-Perot cavities in the arms
 - » Effectively increase arm length ~100 times.



Interferometers as Gravitational Wave Detectors

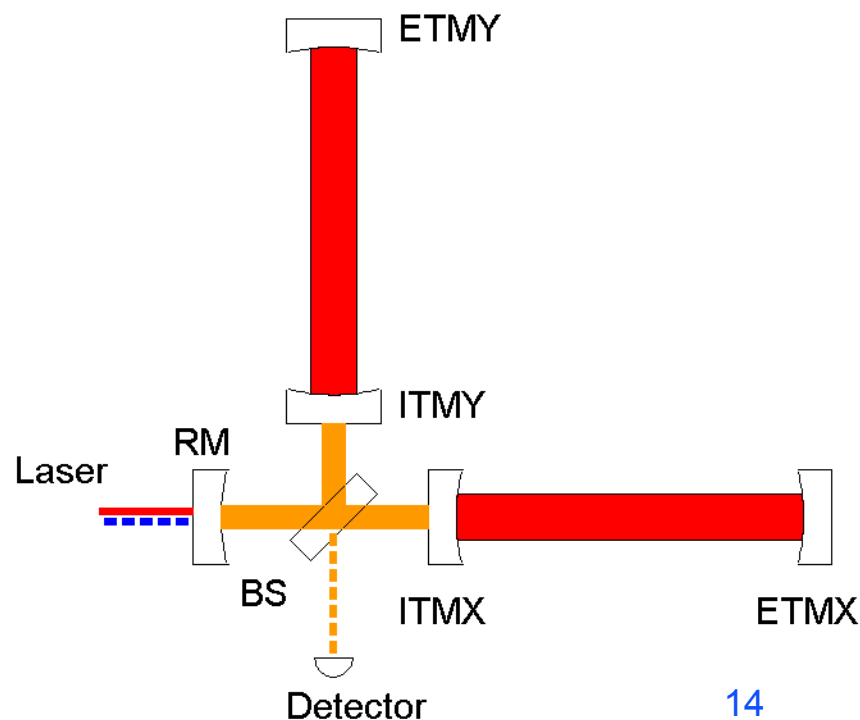
- Gravitational wave effectively stretches one arm while compressing the other.
- Interferometer measures the arm-length difference.
 - » Suspended mirrors act as “freely-falling”.
 - » Dark fringe at the detector.
- Fabry-Perot cavities in the arms
 - » Effectively increase arm length ~100 times.
- Power-recycling mirror
 - » Another factor of ~40 in power.





Back-of-the-Envelope Sensitivity

- Rough sensitivity estimate
 - » Input laser power: ~5 Watt
- Sensitivity (ΔL) $\sim \lambda$ ($\sim 10^{-6}$ m)
 - / Number of Bounces in Arm (~ 100)
 - / Sqrt(Number of Photons ($\sim 10^{21}$))
- $\sim 3 \times 10^{-19}$ m
- Strain Sensitivity:
 - » $h = \Delta L / L \sim 10^{-22}$
 - » $L = 4$ km



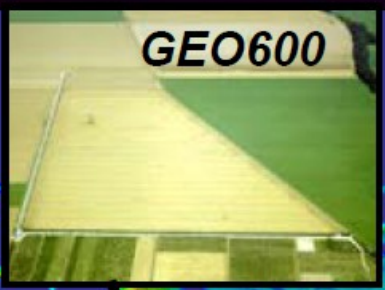


Network of Gravitational-Wave Detectors



*LIGO
Hanford*

4 km
(+2 km)



GEO600

600 m



TAMA, CLIO

300 m
100 m



*LIGO
Livingston*

4 km



VIRGO

3 km

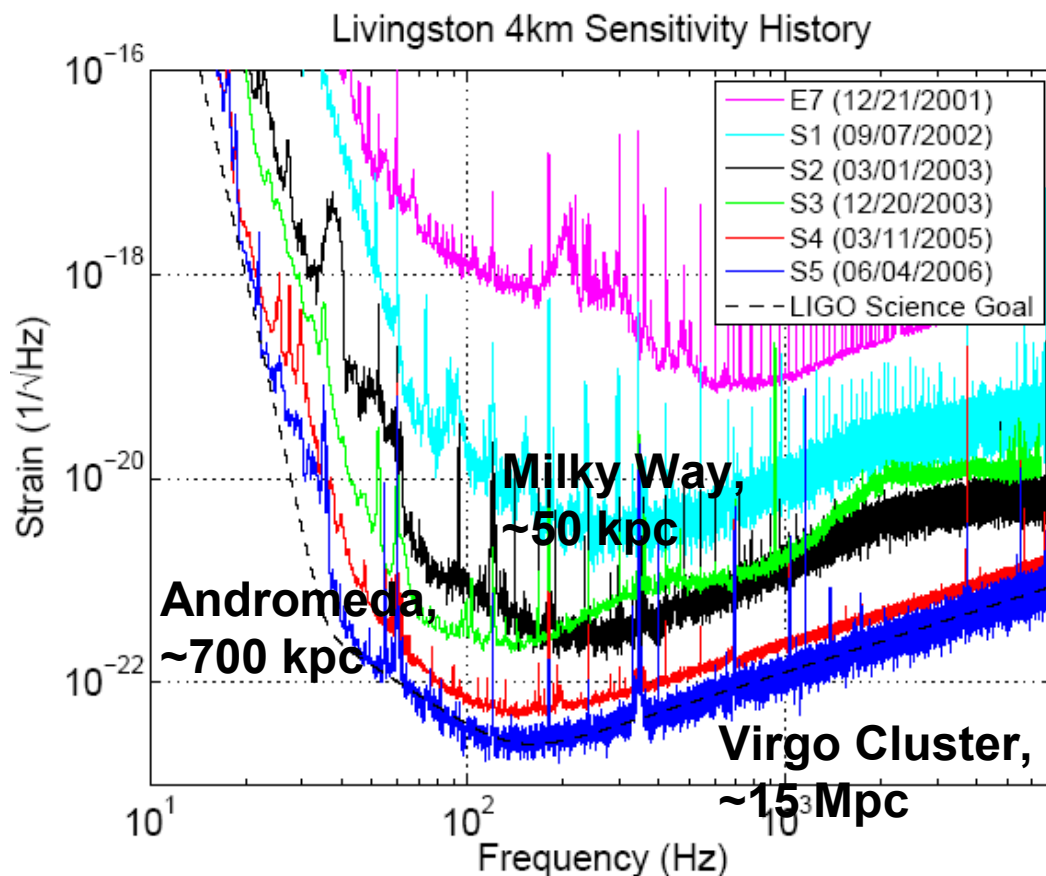


Bars



Sensitivity History

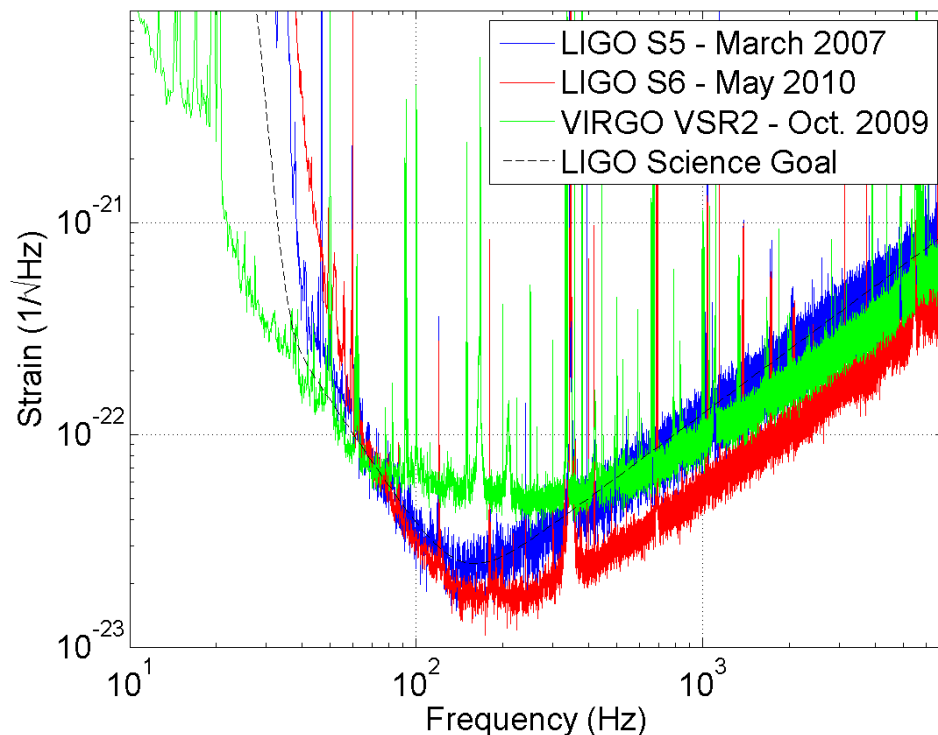
- Substantial sensitivity improvements:
 - » 5 orders of magnitude in ~5 years.
- LIGO reached its design sensitivity in Nov. 2005.
- Science run at design sensitivity (S5) completed in Oct. 2007.
 - » 1 year of H1-L1-H2 coincident time.
- Data analysis still ongoing.
 - » Several results published.





Enhanced LIGO

- In 2007-2008, upgraded 4km detectors H1 and L1.
 - » More powerful laser.
 - » Seismically isolated output mode cleaner.
 - » New locking scheme.
- In 2009-2010, LIGO performed a new science run (S6) at improved strain sensitivity (Enhanced LIGO).
 - » Much of it in coincidence with Virgo and GEO experiments.





Stochastic Background of Gravitational Waves

- Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} \langle \dot{h}_{ab} \dot{h}^{ab} \rangle$$

- Characterized by log-frequency spectrum:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d \ln f}$$

- Related to the strain power spectrum:

$$S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

- Strain scale:

$$h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{GW}(f)} \left(\frac{100 \text{ Hz}}{f} \right)^{3/2} \text{ Hz}^{-1/2}$$



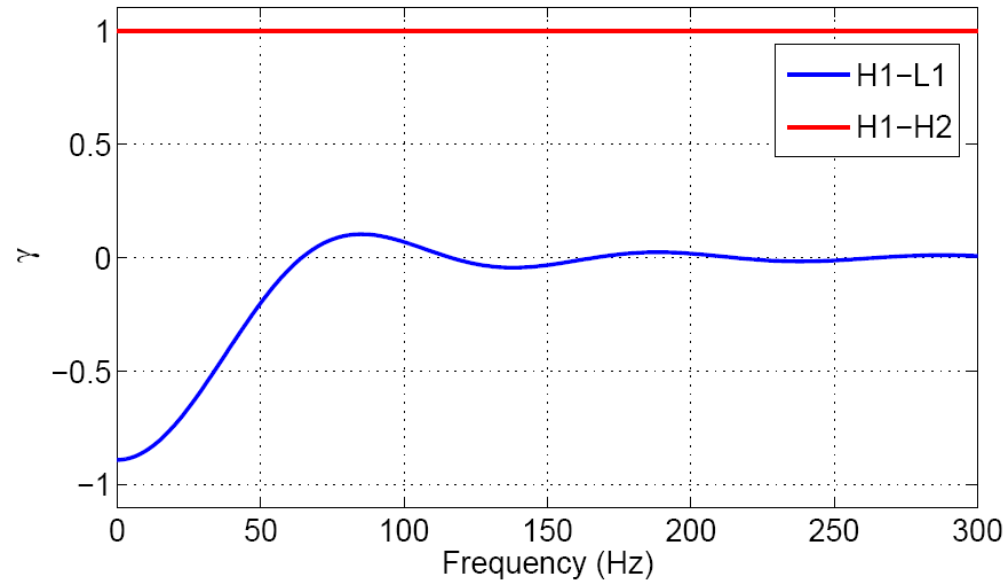
Detection Strategy

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 s_1(t_1) s_2(t_2) Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \tilde{s}_1^*(f) \tilde{s}_2(f) \tilde{Q}(f)$$

Overlap Reduction Function



- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df P_1(f) P_2(f) |\tilde{Q}(f)|^2$$

- Optimal Filter

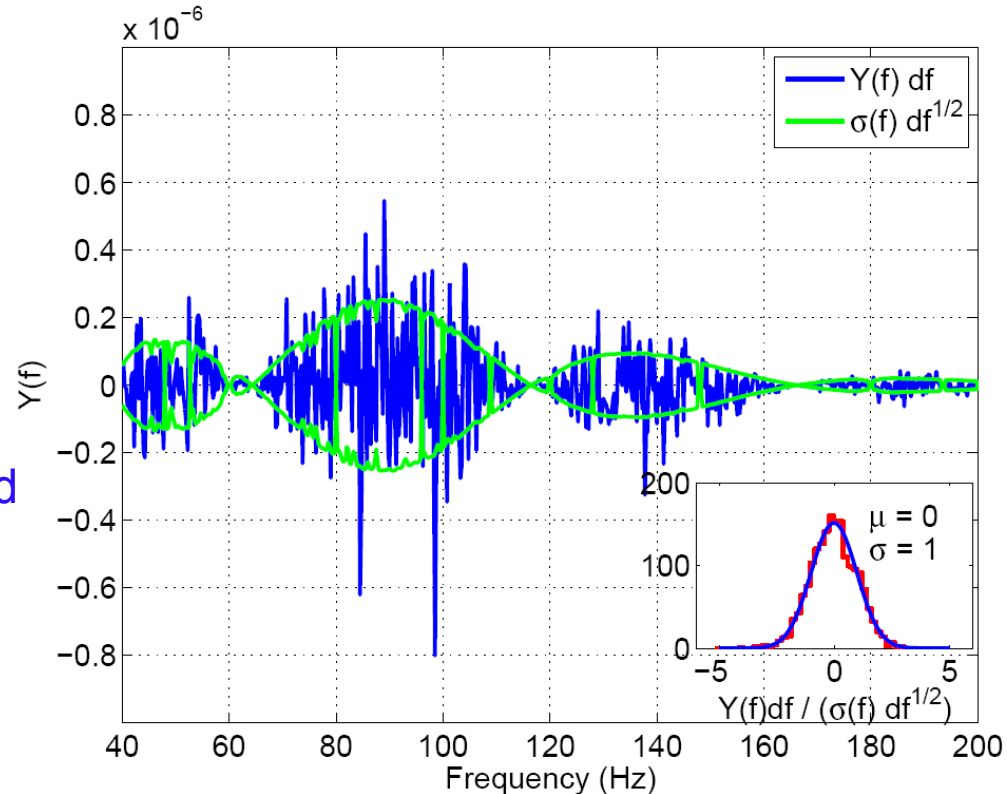
$$\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(f) \Omega_t(f)}{f^3 P_1(f) P_2(f)}$$

For template: $\Omega_t(f) = \Omega_\alpha (f/100 \text{ Hz})^\alpha$

Choose N such that: $\langle Y \rangle = \Omega_\alpha T$

S5 Result

- Use the entire S5 data-set.
 - » 292 days of effective observing time.
- S5 LHO-LLO result:
 - $\Omega_0 \pm \sigma_\Omega = (2.1 \pm 2.7) \times 10^{-6}$
 - » $H_0 = 72 \text{ km/s/Mpc}$
- The frequency band is selected to include 99% of sensitivity, as measured by the integrand of σ^{-2} .
 - » **41.5-169.25 Hz**
- Bayesian 95% UL:
 - » Prior on Ω_0 : S4 Posterior
 - » Marginalize over calibration uncertainties
 - » 95% UL: **6.9×10^{-6}**
- Beginning to constrain models of stochastic GW background.



Abbott et al, Nature 460, 990 (2009)



BBN and CMB Indirect Bounds

- Big-Bang Nucleosynthesis model and observations constrain the total energy at the time of BBN:

$$\Omega_{\text{BBN}} = \int \Omega_{\text{GW}}(f) d(\ln f) < 1.1 \times 10^{-5} (N_{\nu} - 3)$$

- Similar bound is derived from CMB observations.
- In the LIGO frequency band, this becomes:
 - » $\Omega_0^{\text{BBN}} < 1.0 \times 10^{-5}$
 - » $\Omega_0^{\text{CMB}} < 9.5 \times 10^{-6}$
- We have now surpassed these bounds.
 - » Important LIGO milestone!



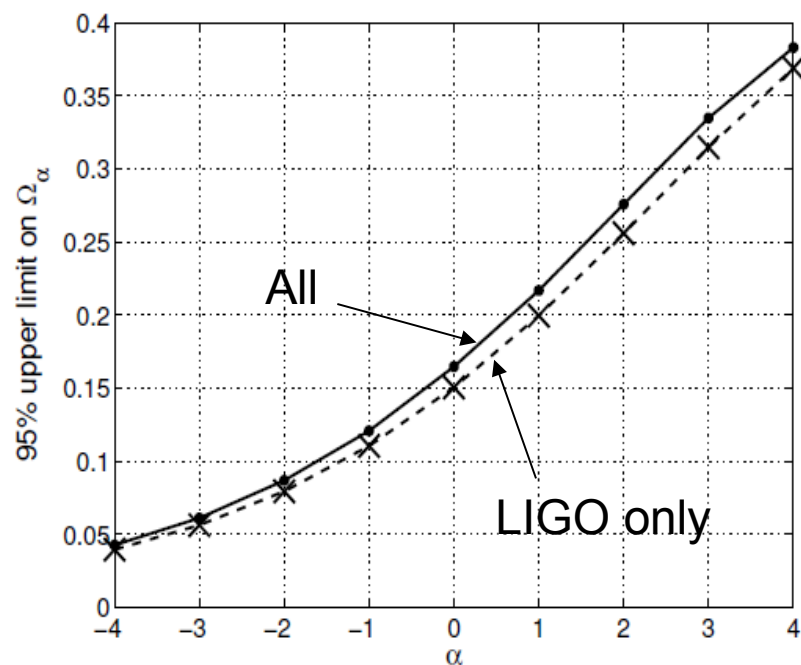
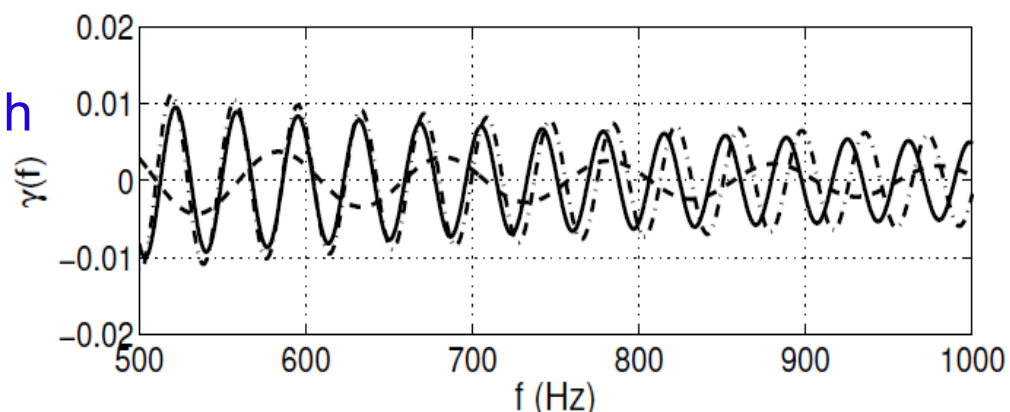
LIGO-Virgo High-Frequency Result

- Repeated a similar analysis at high frequencies: 600-900 Hz.

arXiv:1112.5004, to appear in PRD

- Data from LIGO S5 and Virgo VSR1 runs.
- Using all non-collocated LIGO-Virgo detector pairs.
- Overlap reduction is substantial, but still produced most sensitive measurement in this frequency band.

$$\Omega_0 < 0.16 \text{ (H1-H2-L1-V1)}$$





Anisotropic Searches

- Measure from where (on the sky) the signal comes from.
 - » Time-delay between two detectors.
 - » Earth rotation breaks degeneracies for permanent signals.
- Redefine energy density:

$$\Omega_{\text{GW}}(f) \equiv \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df} = \frac{2\pi^2}{3H_0^2} f^3 \int_{S^2} d\hat{\Omega} \mathcal{P}(f, \hat{\Omega})$$

- Point source (radiometer) search: $\mathcal{P}(\hat{\Omega}) \equiv \eta(\hat{\Omega}_0) \delta^2(\hat{\Omega}, \hat{\Omega}_0)$
- Spherical harmonic decomposition (similar to CMB analyses):

$$\mathcal{P}(\hat{\Omega}) \equiv \sum_{lm} \mathcal{P}_{lm} Y_{lm}(\hat{\Omega})$$

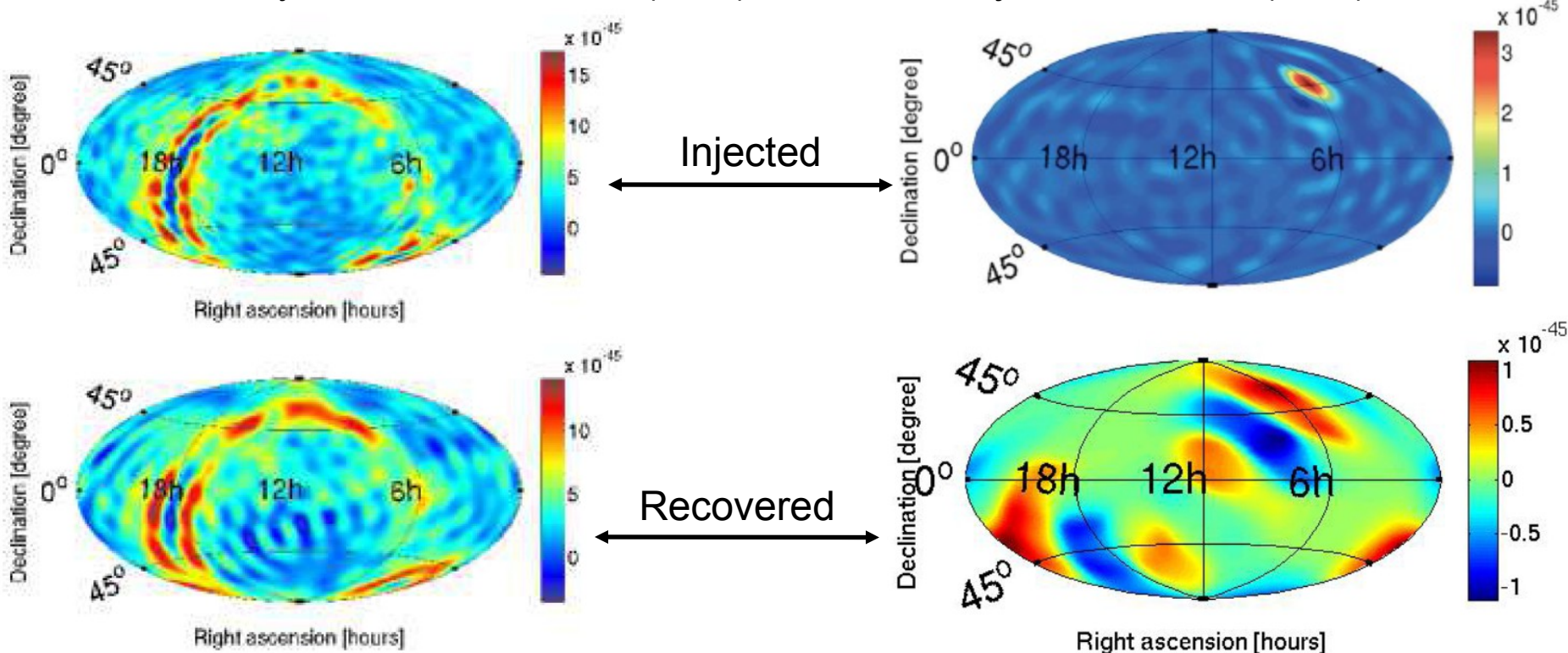


Anisotropic Signal Simulations

Anisotropic stochastic signal added to the data (in software or hardware) and successfully recovered.

WMAP map added to data in software
E. Thrane et al, Phys. Rev. D 80, 122002 (2009).

Point source simulation in hardware
M. Pihlaja's M.S. Thesis (2011).





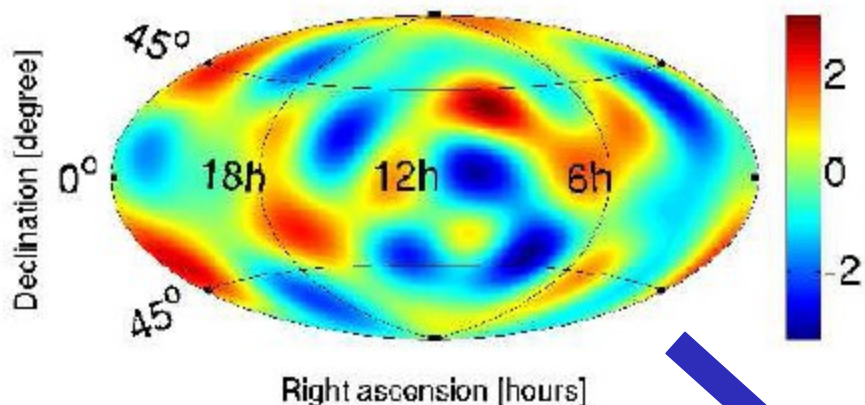
Anisotropic Searches with LIGO S5 Data

SNR Map

Spherical Harmonics Search

Template: $\Omega_{\text{GW}} = \text{const}$

Maximum SNR significance: 25%

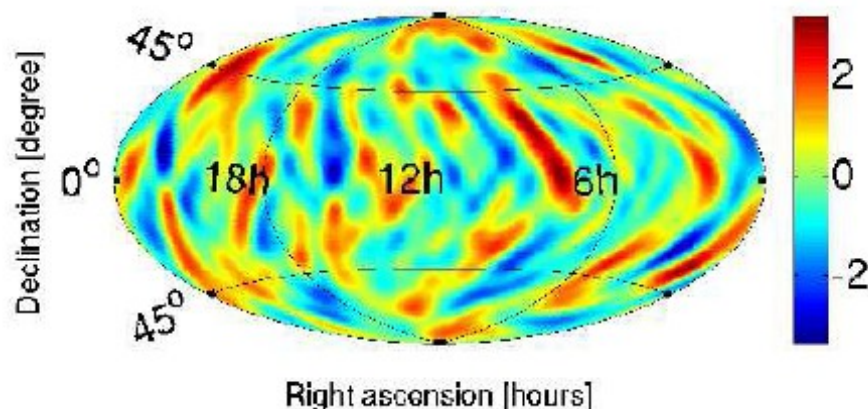


SNR Map

Radiometer Search

Template: strain = const

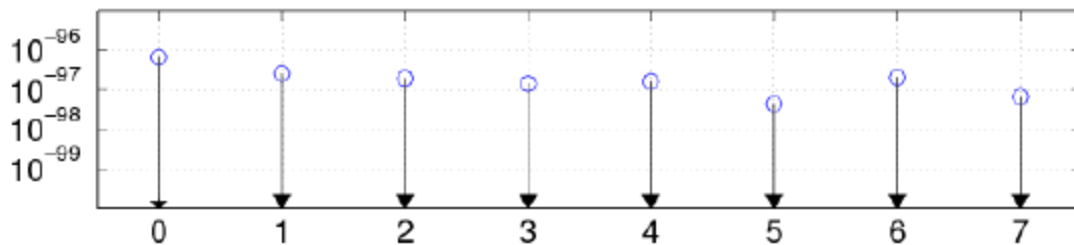
Maximum SNR significance: 53%



90% CL Upper Limit on C_l

$$C_l \approx \frac{1}{2l+1} \sum_m |P_{lm}|^2$$

C_l (strain⁴Hz⁻²sr⁻²)

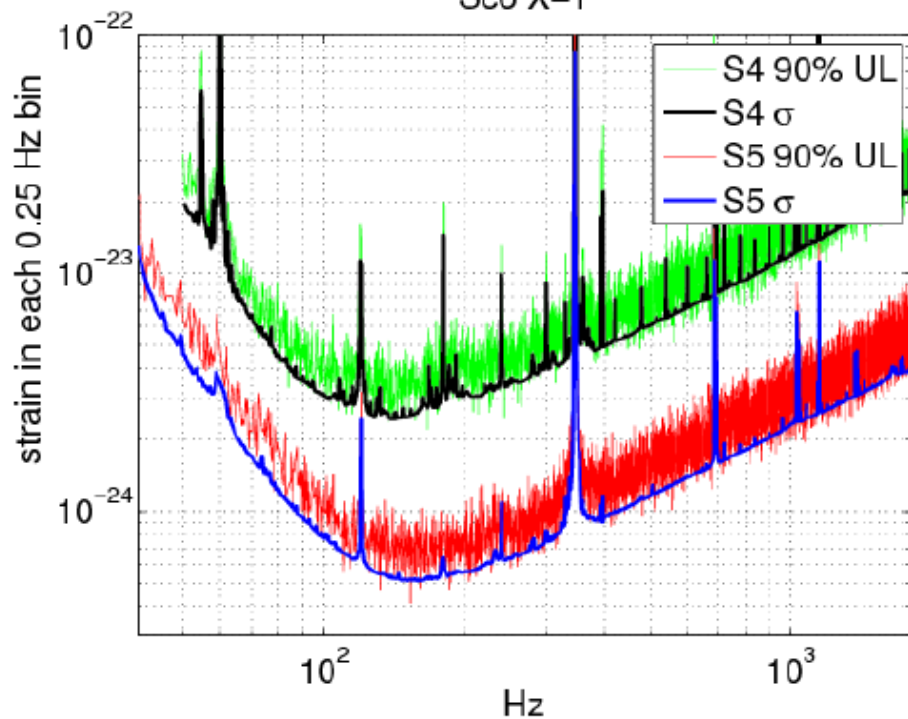




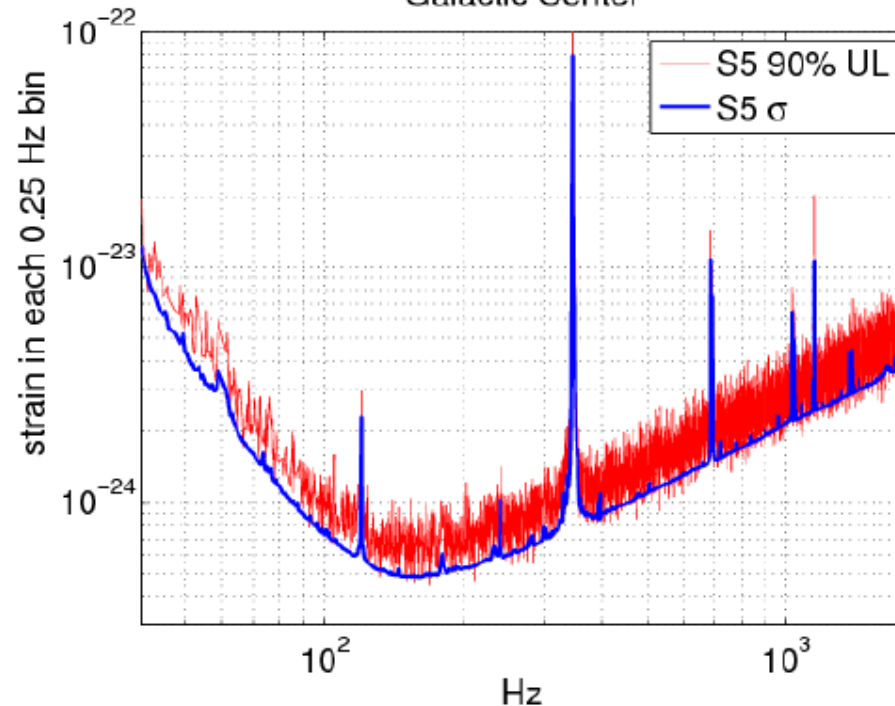
Point-Source Searches with LIGO S5 Data

Can also compute 90% CL upper limits on the strain from specific point-sources in the sky.

Sc0 X-1



Galactic Center

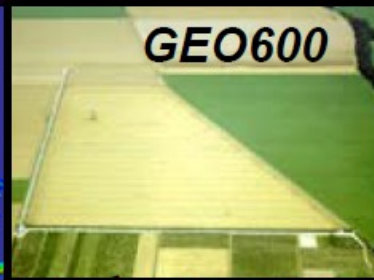




Advanced Detector Era Network in 2015?



*Advanced LIGO
Hanford*



GEO600



LCGT



*Advanced LIGO
Livingston*



Advanced VIRGO



Advanced LIGO

4 km

4 km

600 m

3 km

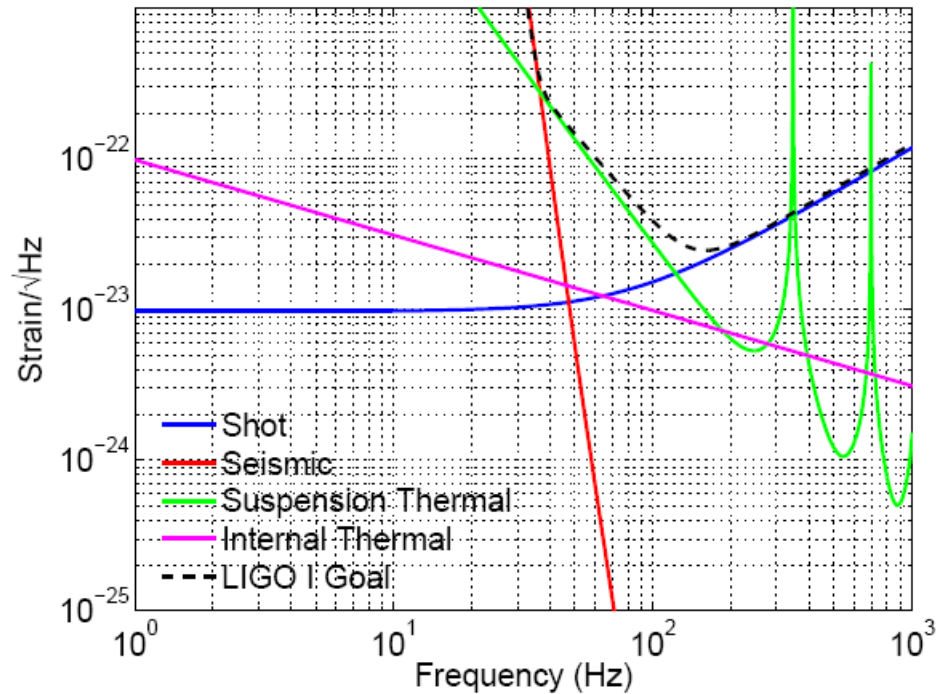
3 km

4 km ?



Advanced LIGO

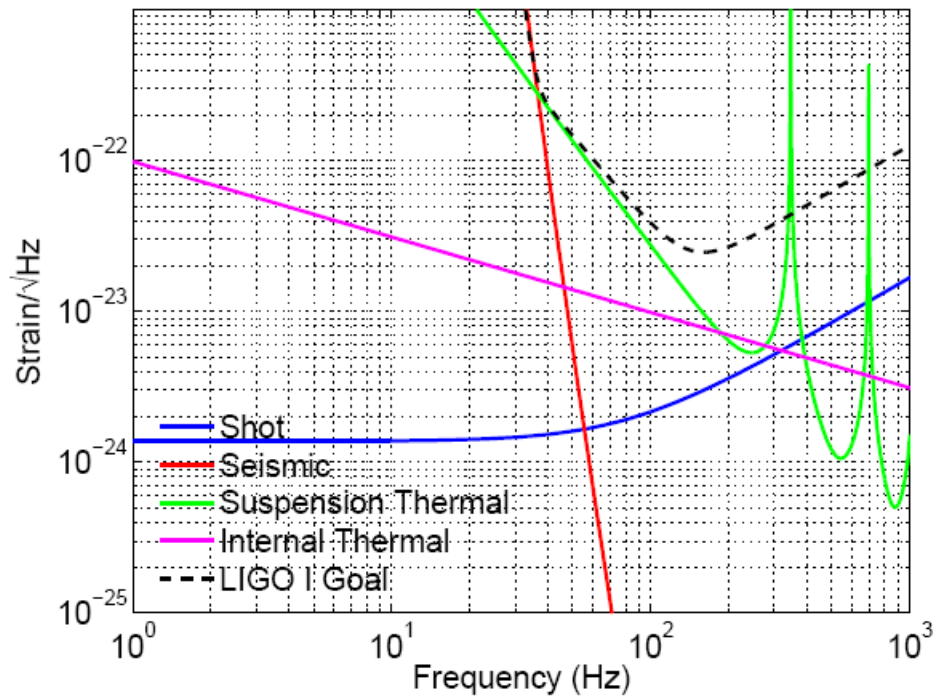
- Keep the same facilities, but redesign all subsystems.
 - » Improve sensitivity over the whole frequency range.





Advanced LIGO

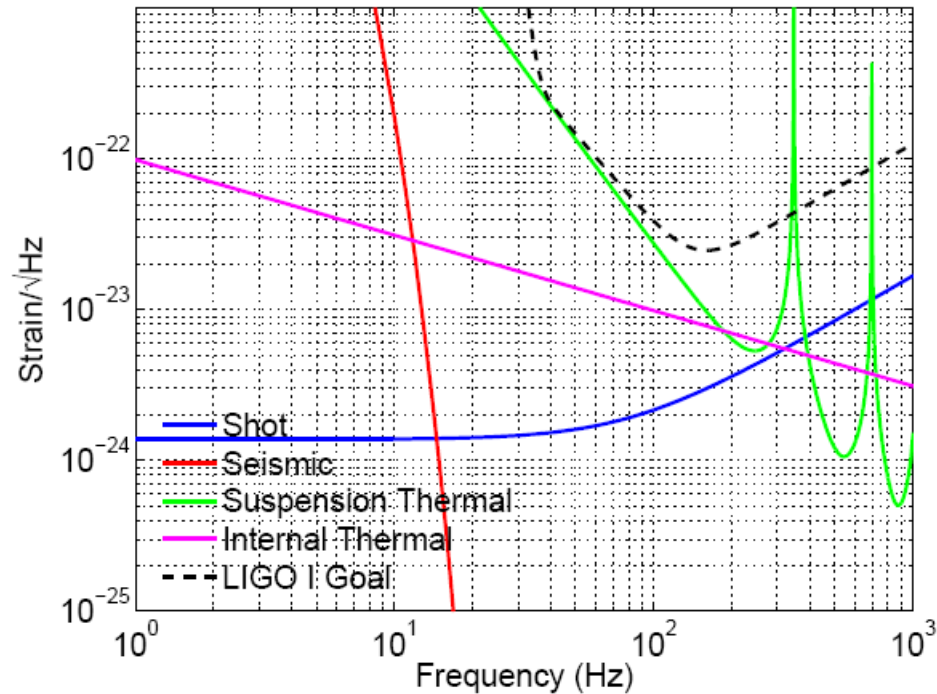
- Keep the same facilities, but redesign all subsystems.
 - » Improve sensitivity over the whole frequency range.
- Increase laser power in arms.





Advanced LIGO

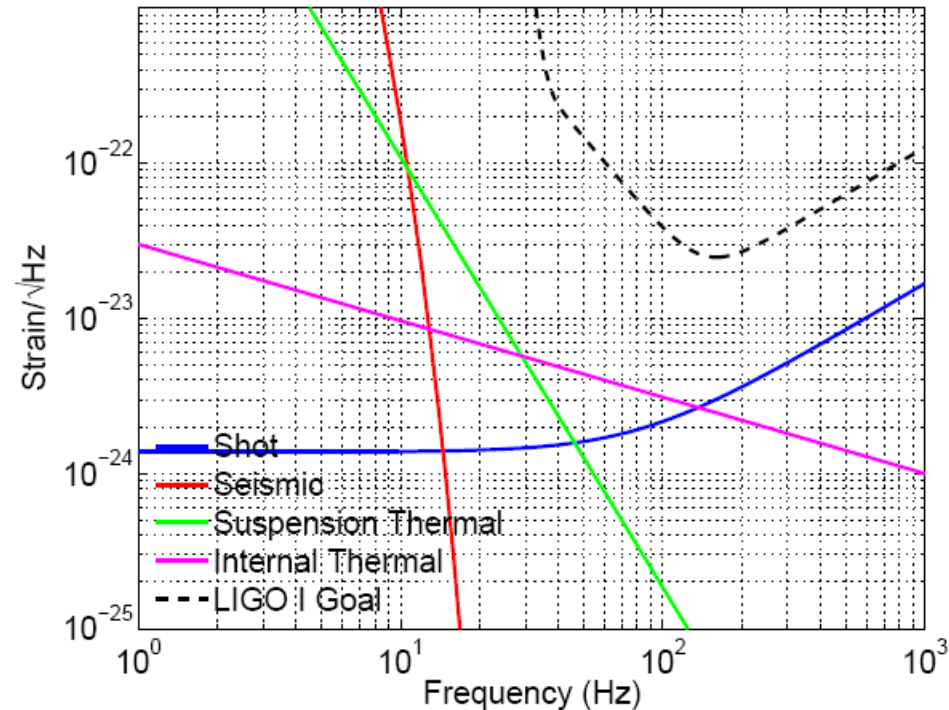
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- Better seismic isolation.
 - » Quadruple pendula for each mass





Advanced LIGO

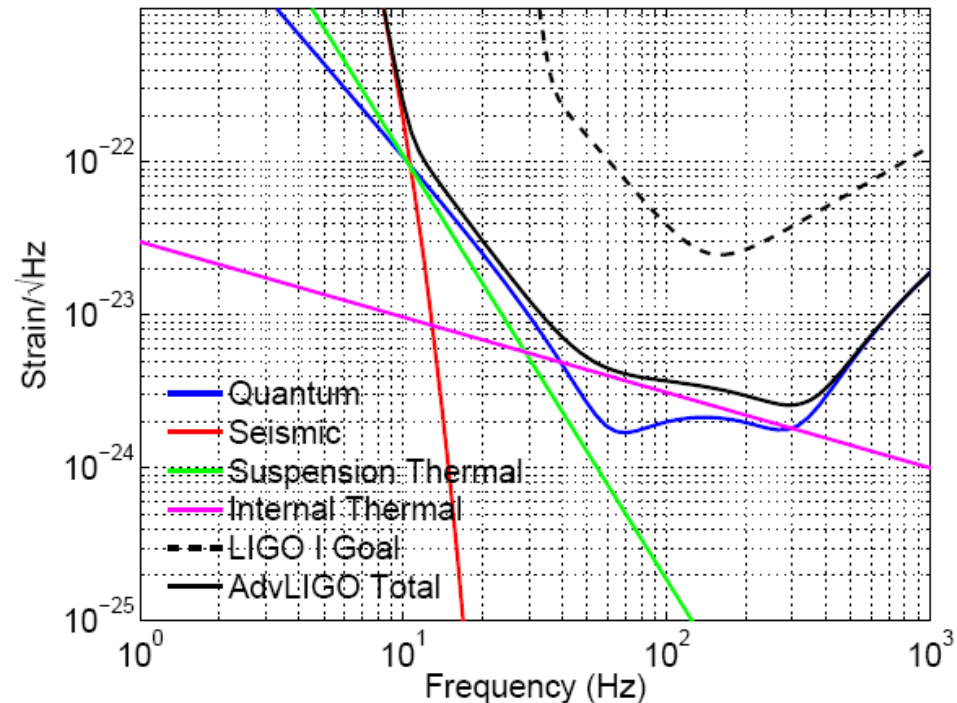
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- Larger mirrors to suppress thermal noise.
- Silica wires to suppress suspension thermal noise.





Advanced LIGO

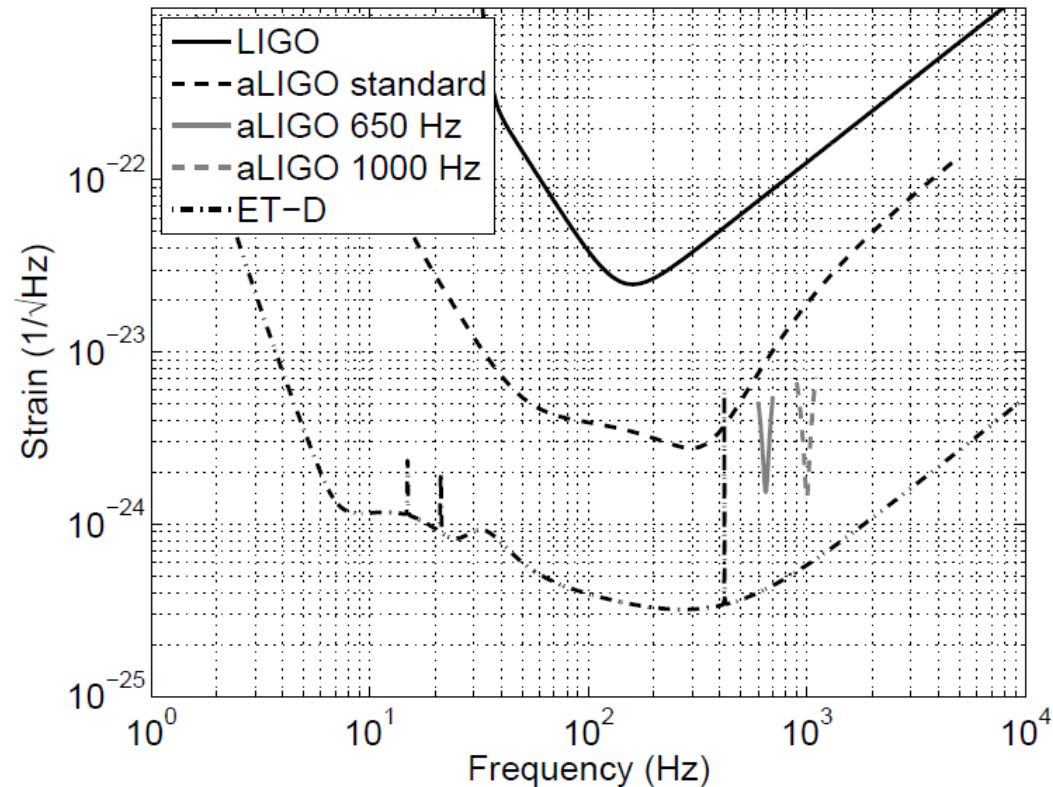
- Keep the same facilities, but redesign all subsystems.
 - » Improve sensitivity over the whole frequency range.
- Increase laser power in arms.
- Better seismic isolation.
 - » Quadruple pendula for each mass
- Larger mirrors to suppress thermal noise.
- Silica wires to suppress suspension thermal noise.
- “New” noise source due to increased laser power: radiation pressure noise.
- Signal recycling mirror
 - » Allows tuning sensitivity for a particular frequency range.





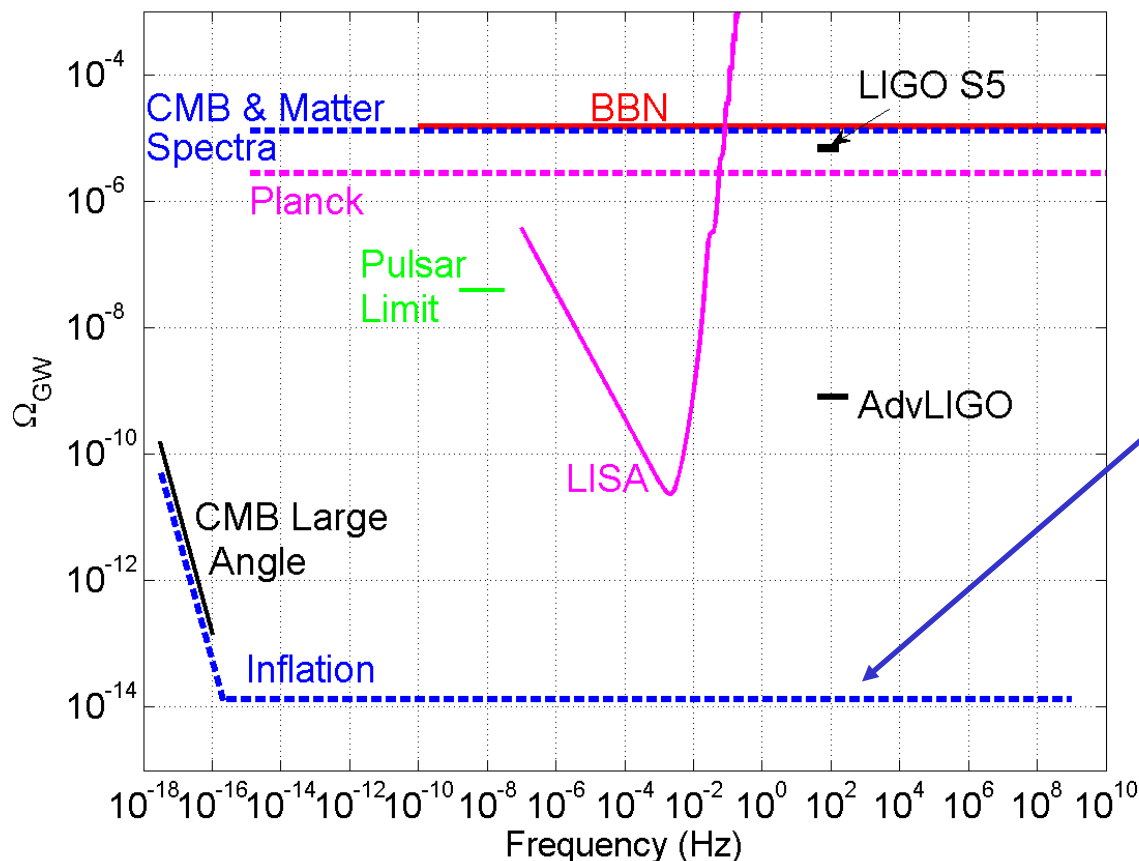
Future of Stochastic Searches

- Second generation:
 - ◆ Advanced LIGO/Virgo, GEO-HF, KAGRA.
 - ◆ First data: 2014/2015.
 - ◆ 10x improvement in strain.
 - ◆ Down to 10 Hz.
 - ◆ Reaching $\Omega_0 \sim 10^{-9}$.
- Third generation:
 - ◆ Einstein Telescope design study.
 - ◆ Another 10x improvement in strain, down to few Hz.
 - ◆ Reaching $\Omega_0 \sim 10^{-12}$.





Advanced Detectors and the Stochastic Background

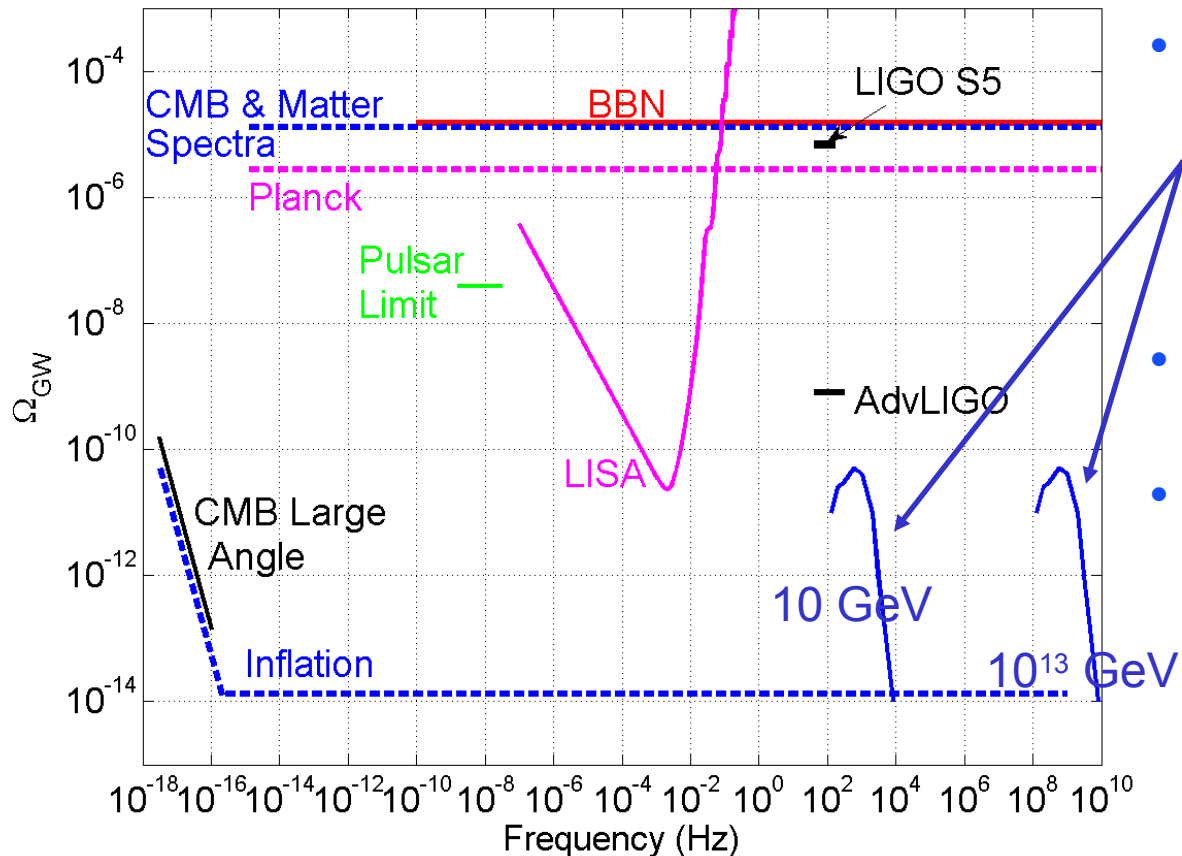


- Standard inflationary models are weakly dependent on frequency.
- Out of reach of advanced detectors by ~5 orders of magnitude

Catalog of models:
<http://homepages.spa.umn.edu/~gwplotter/>



Advanced Detectors and the Stochastic Background



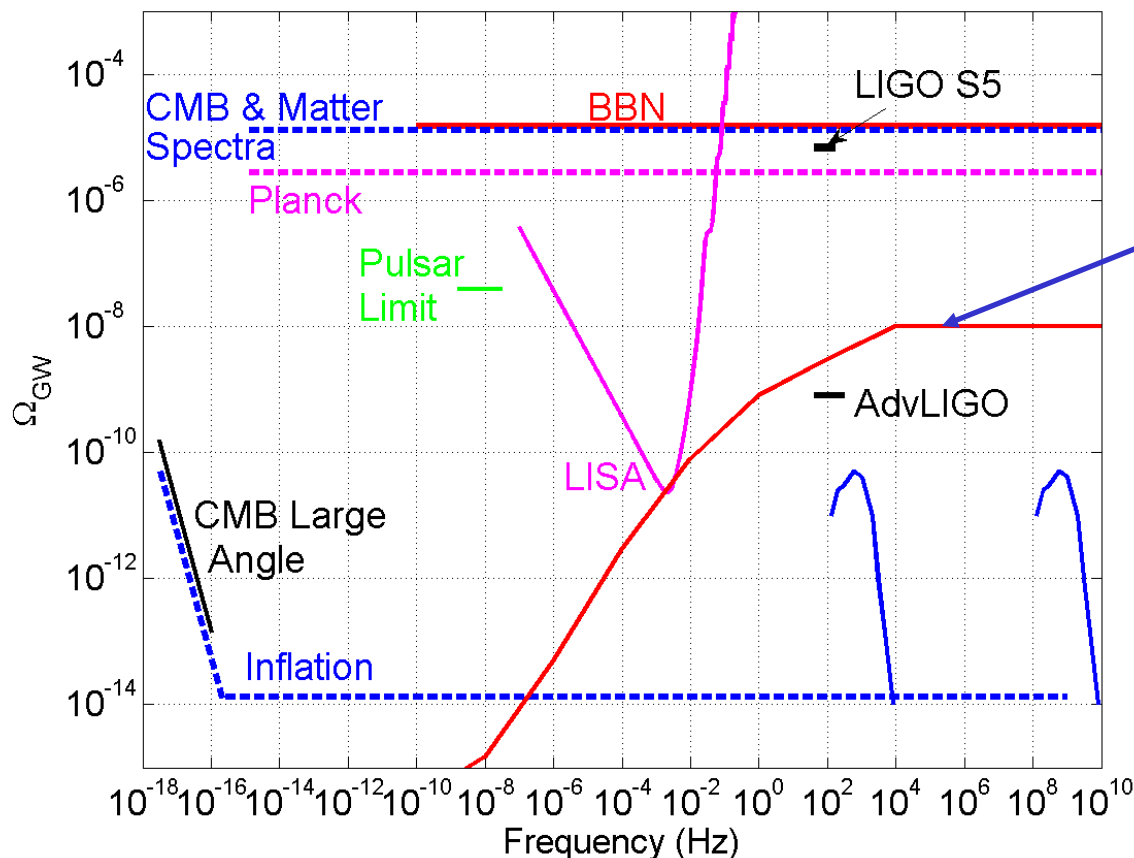
- If inflation ends with a preheating resonant phase, inflaton energy is efficiently transferred to other particles.
- Can have significant increase in GW background.
- Peak depends on energy scale.
 - » Easter & Lim, JCAP 0604, 010 (2006).
 - » Easter et al, PRL 99, 221301 (2007).
 - » Easter, Nucl. Phys. Proc. Suppl. 194, 33 (2009).

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Advanced Detectors and the Stochastic Background

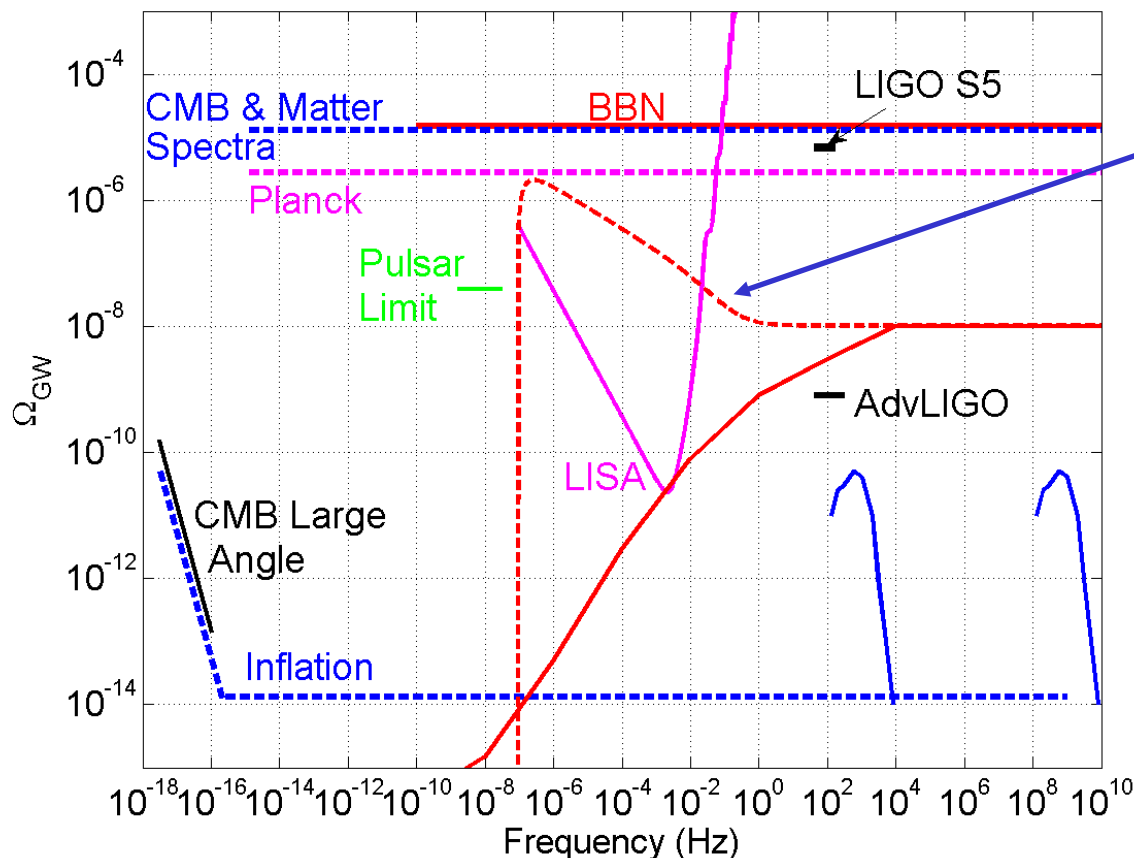


- Axion-based inflation models include axion-gauge couplings.
- Gauge backreaction on the inflaton extends inflation.
- This late inflationary phase increases GW production at high frequencies.
 - » Barnaby et al, arXiv:1110.3327.

Catalog of models:
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Advanced Detectors and the Stochastic Background



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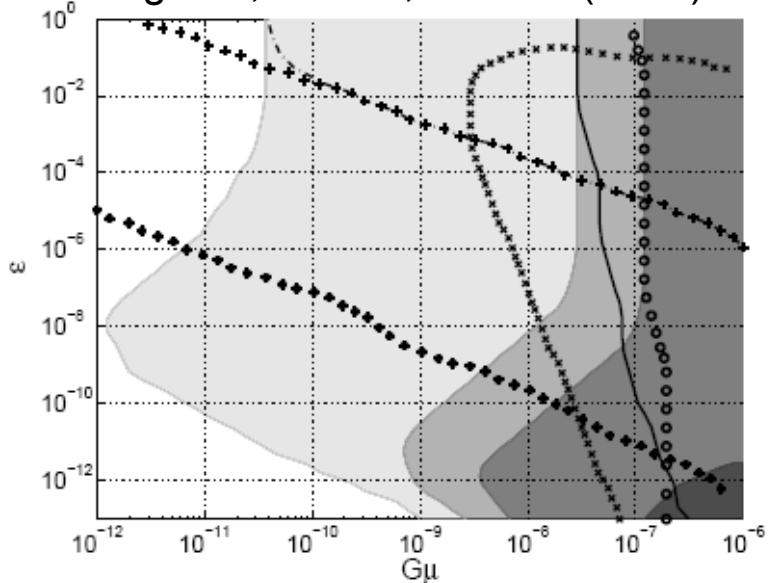
- Cosmic (super)strings models: cusps or kinks moving at relativistic speeds produce bursts of gravitational radiation.
- Integrating over the whole universe leads to a GW background.
- Large parameter space, some of it already probed by initial LIGO.
 - » Damour & Vilenkin, PRL 85, 3761 (2000).
 - » Siemens et al, PRL 98, 111101 (2007).
 - » Olmez et al, PRD 81, 104028 (2010).



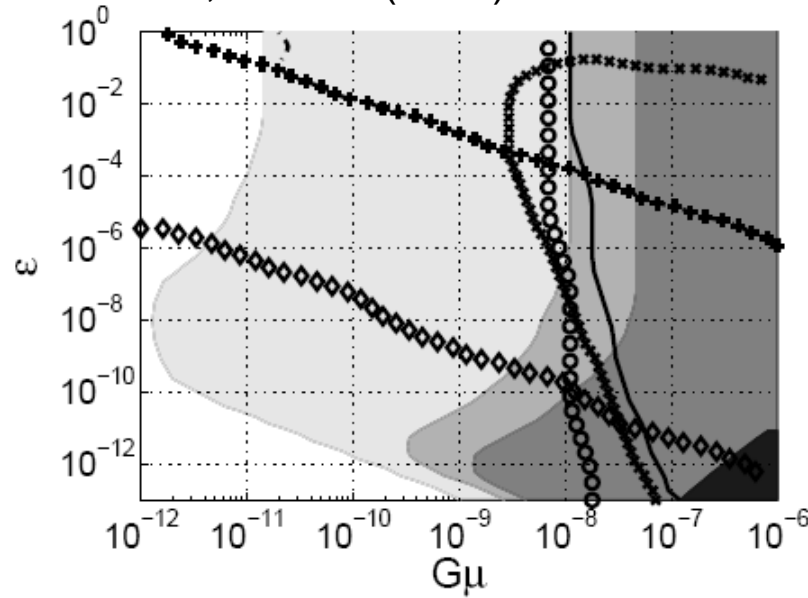
Systematic Study: Cosmic (Super)Strings

- String cusps or links moving at the speed of light produce GW bursts.
- Integrating over the entire universe gives a stochastic background.
- Parameters (small-loop scenario):
 - » loop-size parametrized by: $10^{-13} < \epsilon < 1$
 - » String tension: $10^{-12} < G\mu < 10^{-6}$
 - » Reconnection probability: $10^{-3} < p < 1$

Cusps: X. Siemens, V.M., J. Creighton, PRL98, 111101 (2007).

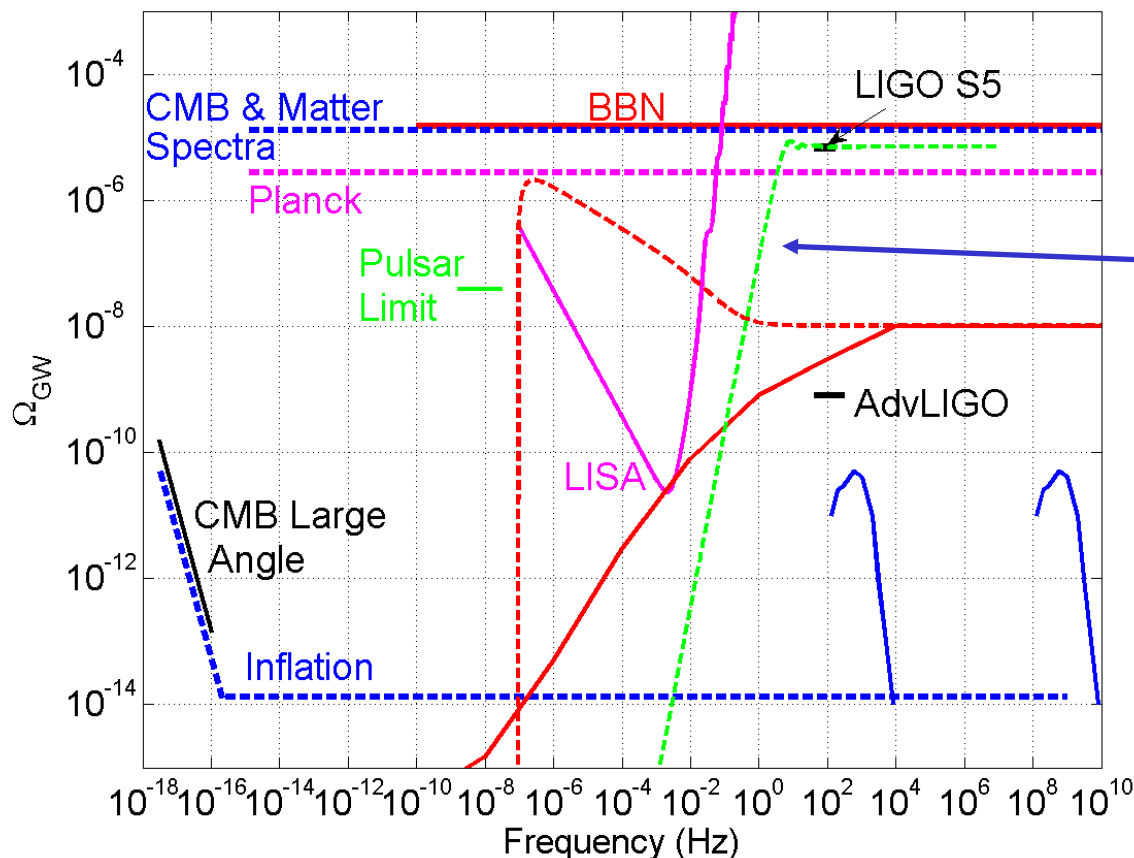


Kinks: S. Olmez, V.M., X. Siemens, PRD81, 104028 (2010).





Advanced Detectors and the Stochastic Background

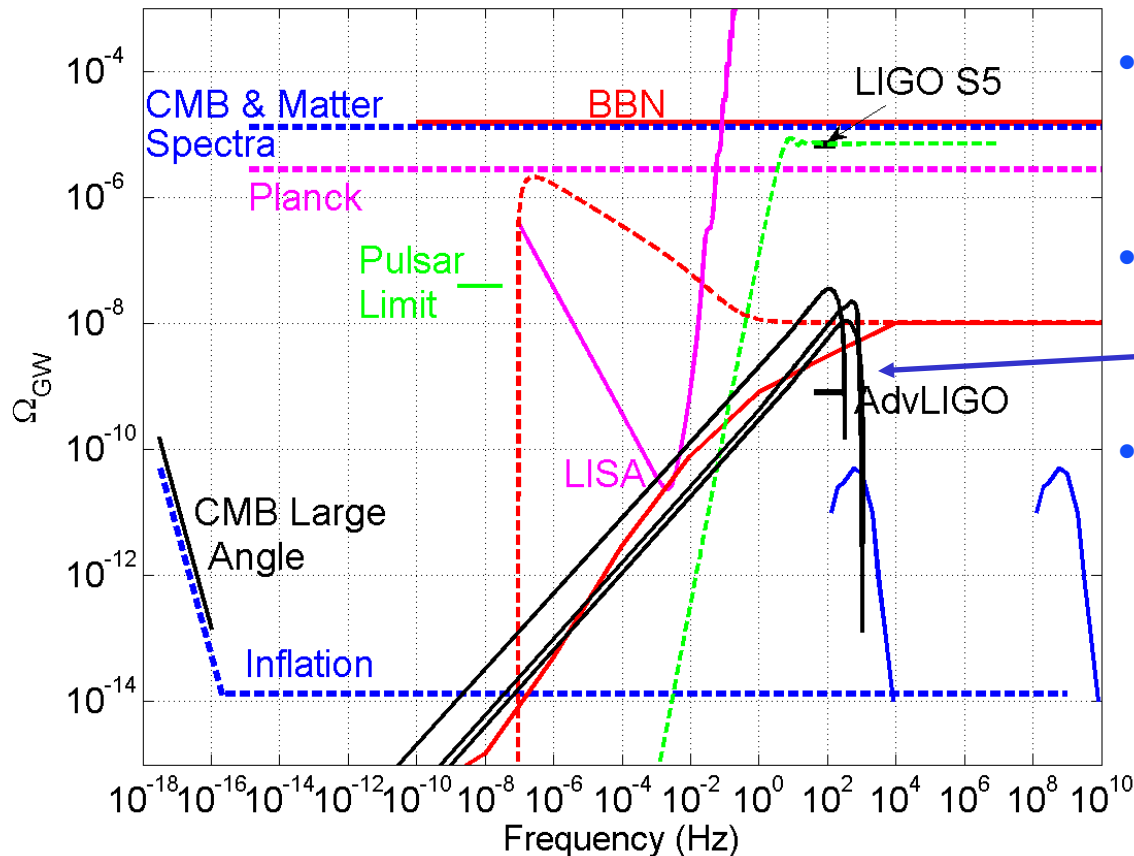


- Alternative cosmologies, such as pre-Big-Bang models, can lead to strong GW backgrounds at high frequencies.
 - » Gasperini & Veneziano, Phys. Rep. 373, 1 (2003).
 - » Buonanno et al, PRD 55, 3330 (1997).

Catalog of models:
<http://homepages.spa.umn.edu/~gwplotter/>



Advanced Detectors and the Stochastic Background



- Individual neutron star and/or black hole pairs generate chirp GW signals.
- Integrating over the whole universe ($z < 6$) leads to a GW background.
- Peak in the LIGO band.
 - » Phinney, ApJ 380, L17 (1991).
 - » Ignatiev et al., MNRAS 327, 531 (2001).
 - » Regimbau & de Freitas Pacheco, ApJ 642, 455 (2006).
 - » Wu et al, arXiv:1112.1898.

Catalog of models:
<http://homepages.spa.umn.edu/~gwplotter/>

Binary Coalescences: Model

Energy Density:
$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c c} \int_{z_{\text{inf}}}^{z_{\text{sup}}} R_z(z) \frac{1}{4\pi d_L^2(z)} \frac{dE_{\text{GW}}(f)}{df} dz$$

Energy emitted by a single binary:
$$\frac{dE_{\text{GW}}}{df} = \frac{(G\pi)^{2/3}}{3} (M_c^z)^{5/3} f^{-1/3}$$

Rate of Binaries:
$$R_z(z) = \lambda R_V(z) \frac{dV(z)}{dz}$$

Mass Fraction Parameter λ (indicated by an arrow pointing to the coefficient in the equation above)

Cosmology (indicated by an arrow pointing to the volume element $\frac{dV(z)}{dz}$ in the equation above)

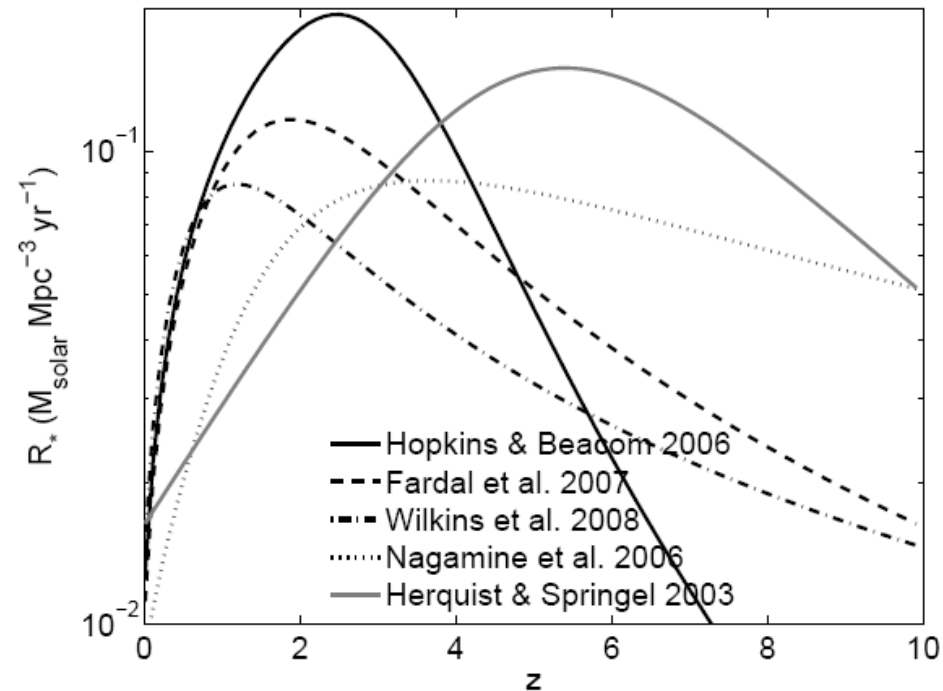
$$R_V(z) = \int \frac{1}{1+z_f} R_*(t_c(z) - t_d) P(t_d) dt_d$$

Star Formation Rate (indicated by an arrow pointing to R_* in the equation above)

Time delay between formation and coalescence (indicated by an arrow pointing to $t_c(z) - t_d$ in the equation above)

Binary Coalescences: Model

- This model has been around for >20 years:
 - Phinney, ApJ 380, L17 (1991).
- Many papers, multiple authors, different tweaks.
- Wu, Mandic, Regimbau, arXiv:1112.1898, to appear in PRD:
 - Systematic study of the accessibility of the model to Advanced detectors.
 - Scan λ - M_c parameter space.
 - Different star-formation rates.
 - Different time-delays.



Population synthesis: $P(t) \sim t^\alpha$, for $t > t_{\min}$:

$$\alpha = -0.5, -1, -1.5$$

$$t_{\min} = 20, 100 \text{ Myr (BNS)}$$

$$t_{\min} = 100, 500 \text{ Myr (BBH)}$$

Short GRBs: log-normal distribution.

No time-delay.



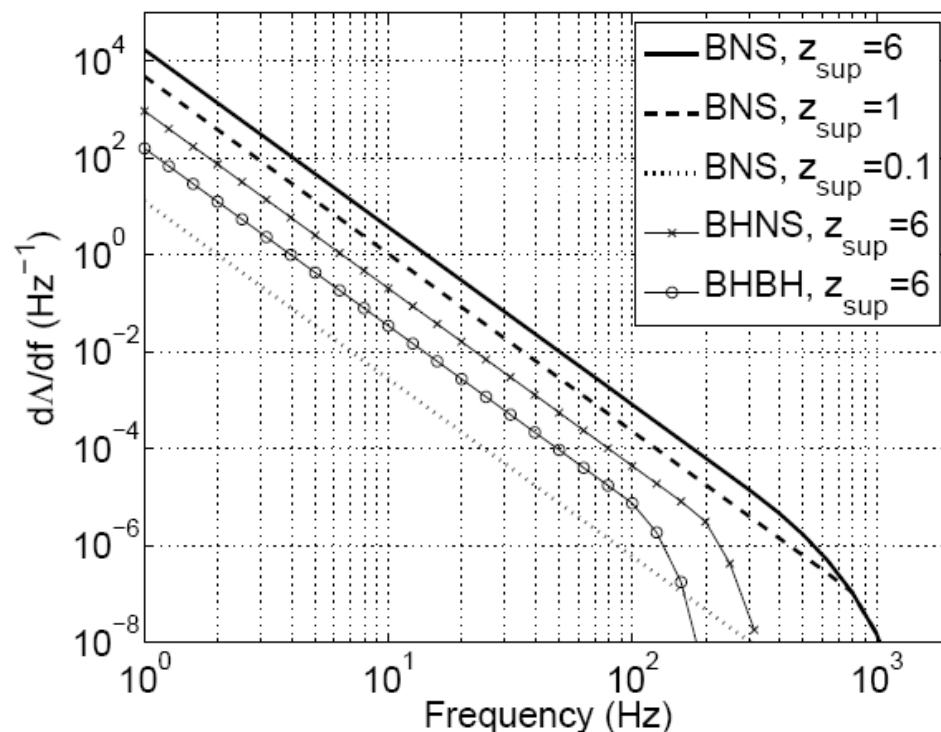
Not a Continuous Background

- ...at most frequencies.

Define duty cycle:

$$\frac{d\Lambda}{df} = \int_0^{z_{\text{sup}}} R_z(z) \frac{d\tau(z)}{df} dz$$

- Popcorn regime, or individual unidentifiable chirps.





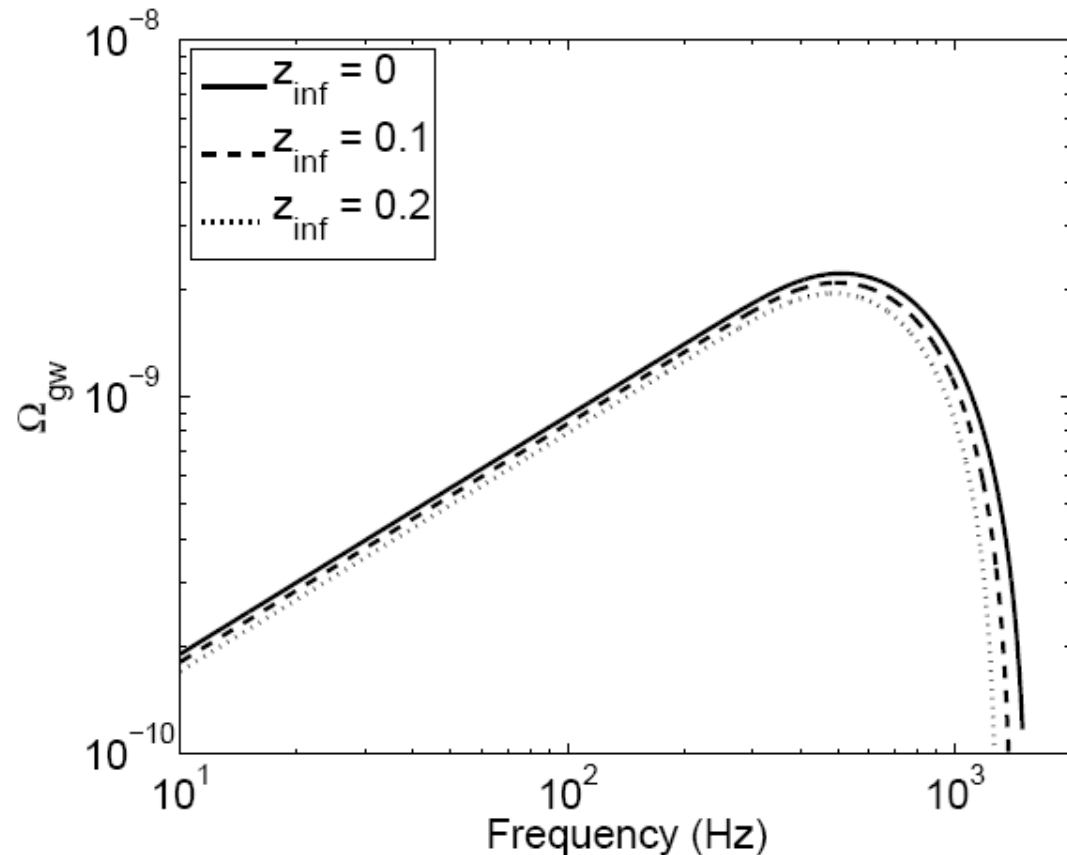
Example Spectra

- Spectrum peaks in the LIGO/Virgo band.

Dominated by the far-away contributions.

Excluding loudest nearby binaries does not change the spectrum significantly.

- Stochastic search pipeline would possibly reject the loudest CBC transients, but would integrate over all unidentifiable chirps.

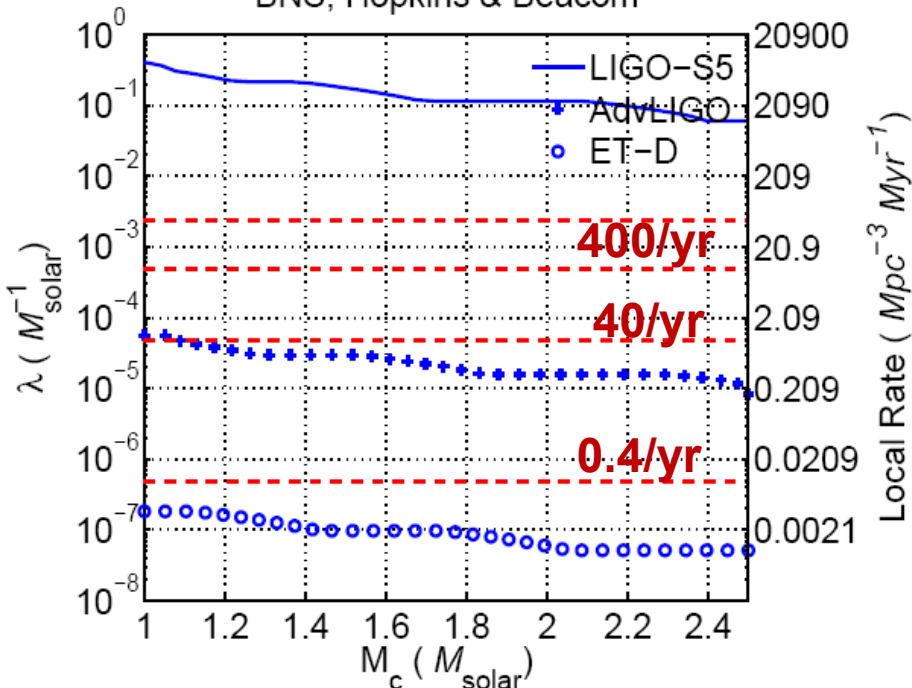




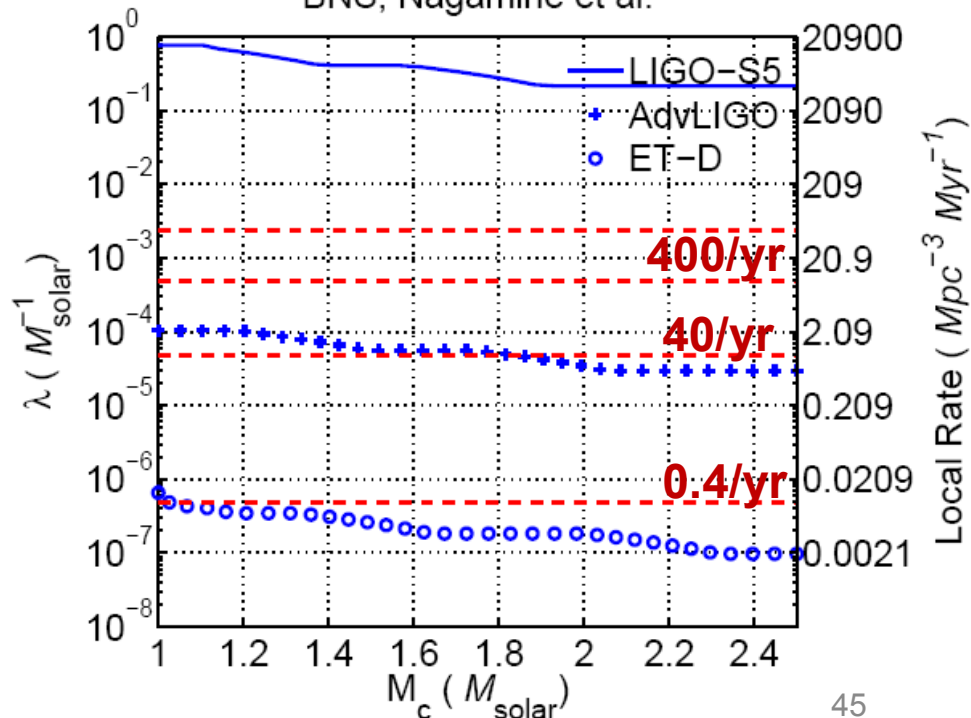
Binary Neutron Stars

- 1) aLIGO should see GW background corresponding to “realistic” coalescence rates.
- 2) Third generation detectors will see this as a “foreground”.
- 3) Star formation rate has little effect.

BNS, Hopkins & Beacom



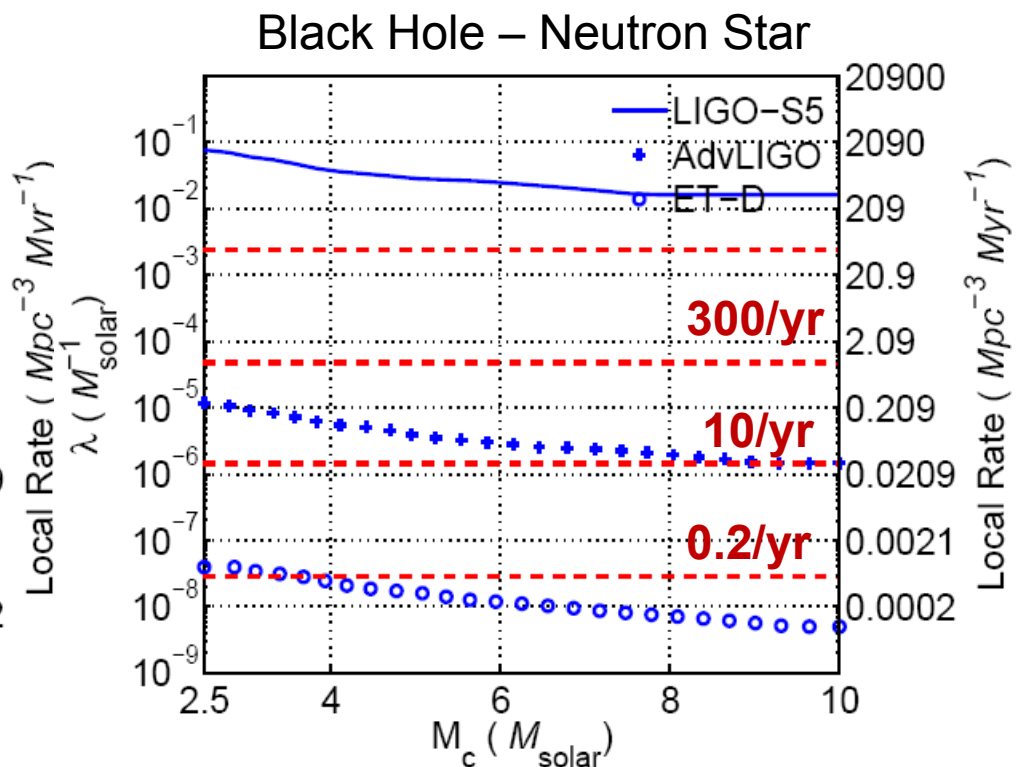
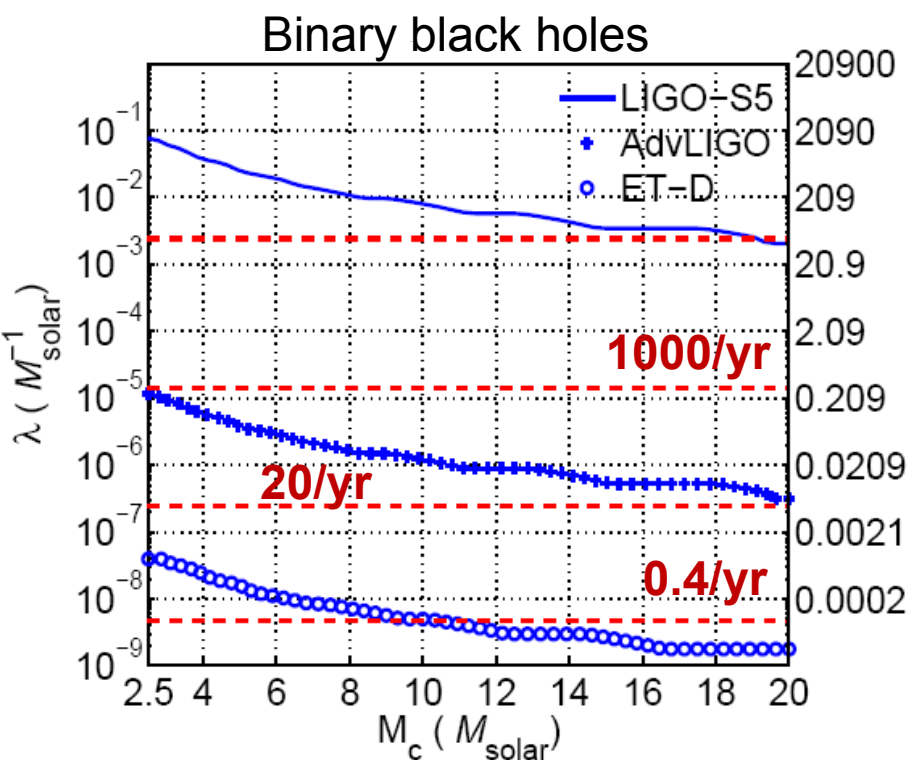
BNS, Nagamine et al.





BBH & BHNS

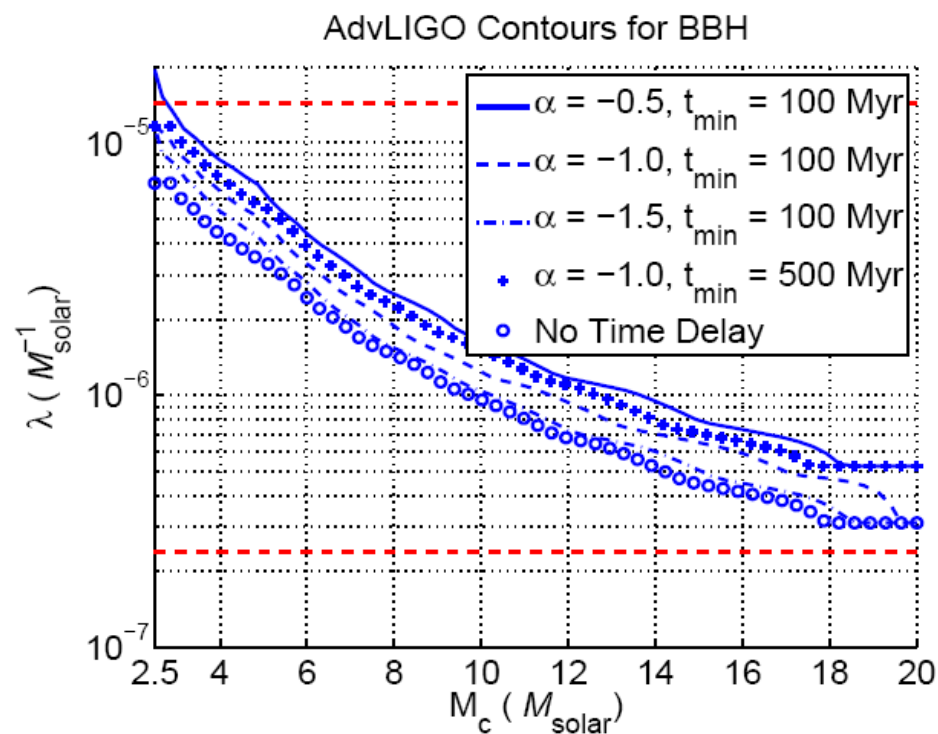
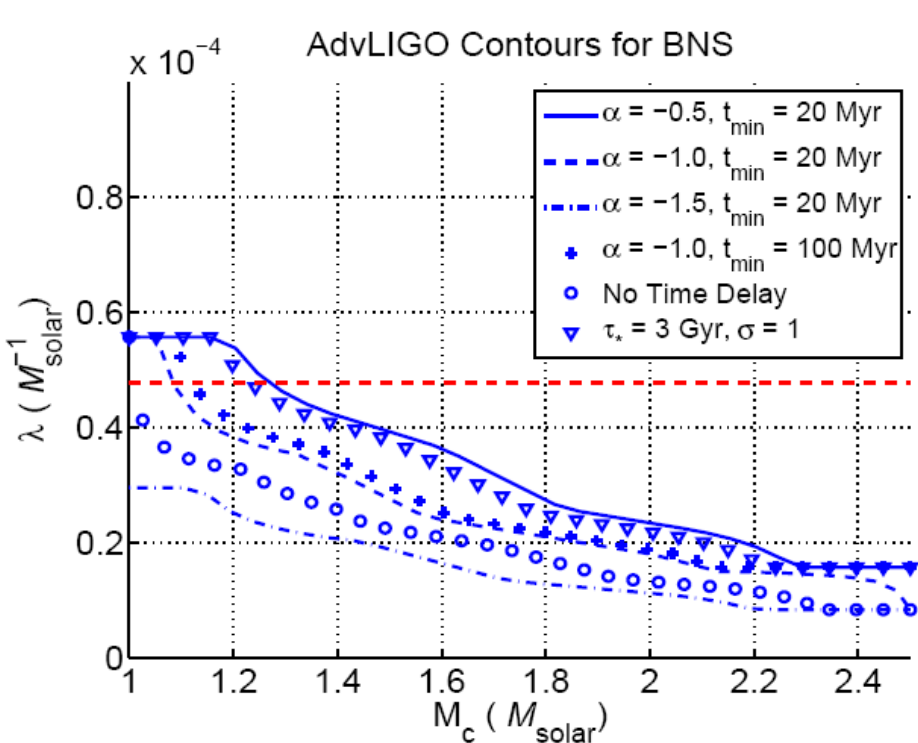
Similar conclusions apply for BBH and BHNS systems.





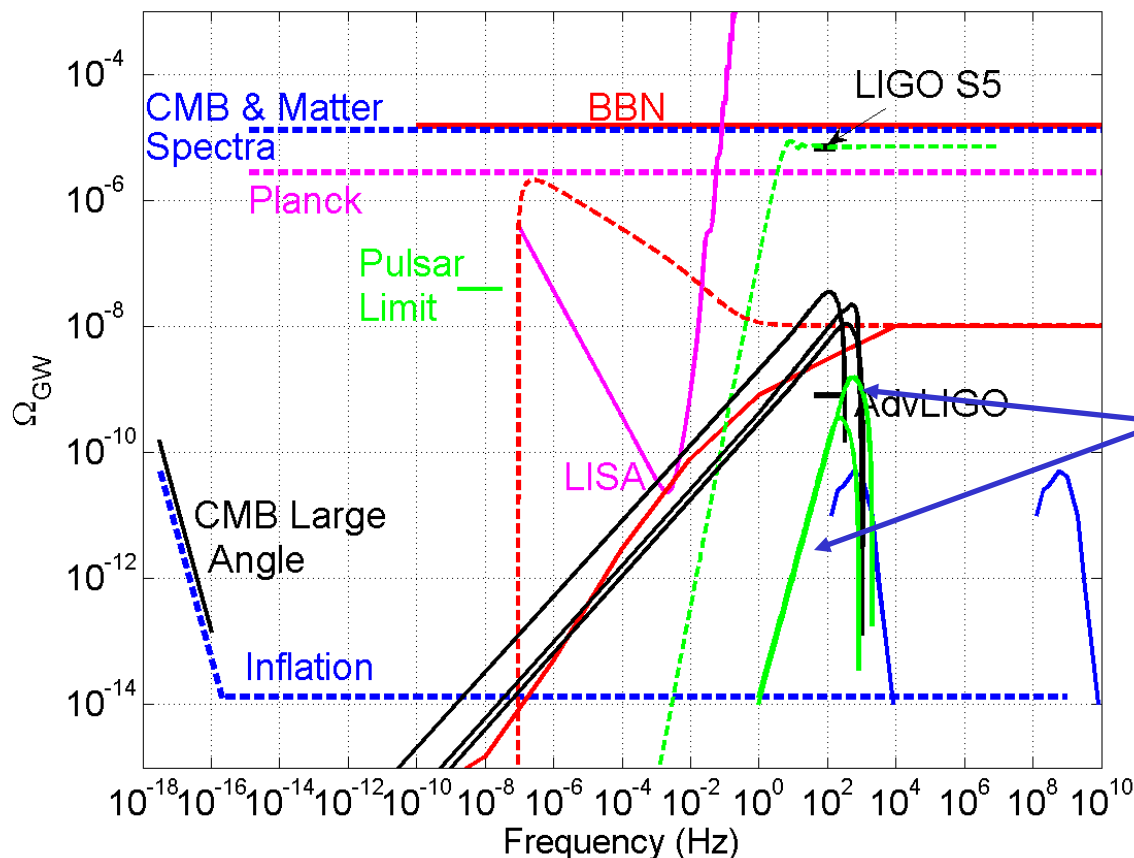
Time-Delay Distribution

Time-delay distribution (functional form and minimum delay) have no qualitative effect.





Advanced Detectors and the Stochastic Background

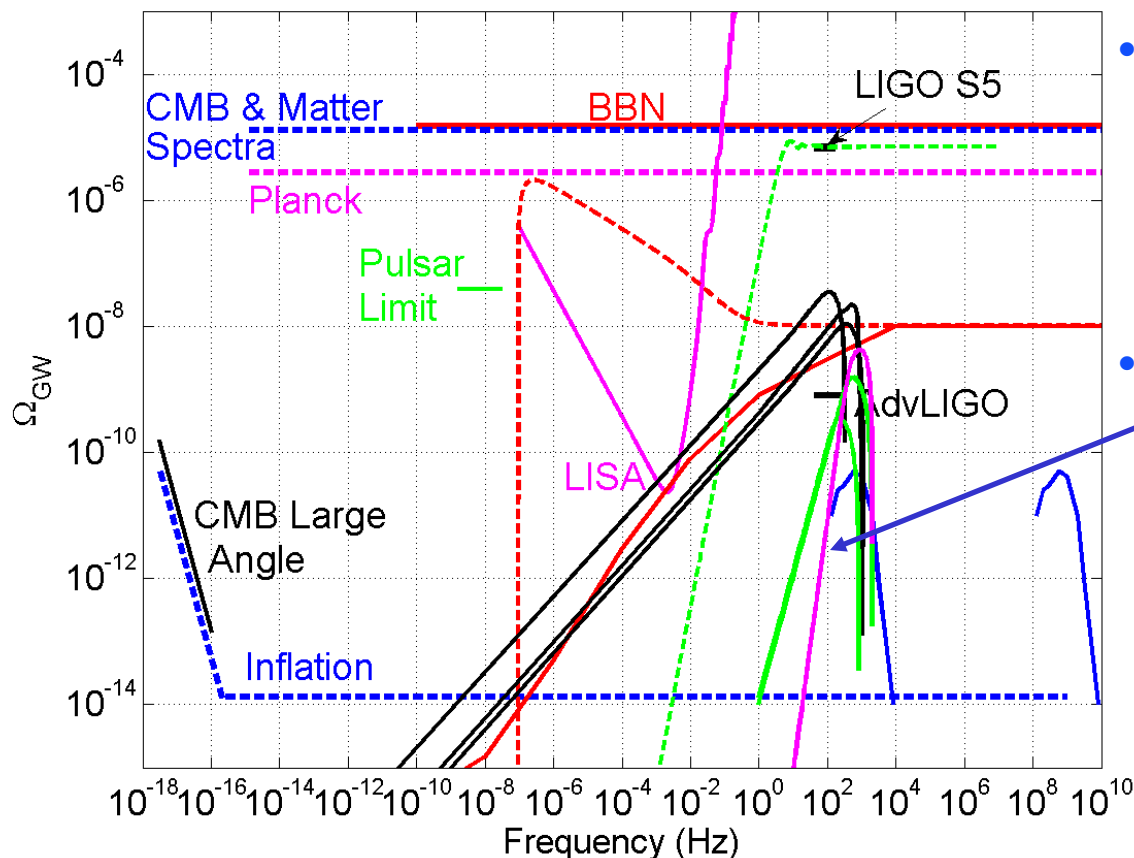


- Neutron stars can have a variety of instabilities: r-modes, bar-modes etc.
- Integrating over the entire universe leads to a GW background.
 - » Owen et al, PRD 58, 084020 (1998).
 - » Lai & Shapiro, ApJ 442, 259 (1995).
 - » Regimbau & de Freitas Pacheco, A&A 376, 381 (2001).

Catalog of models:
<http://homepages.spa.umn.edu/~gwplotter/>



Advanced Detectors and the Stochastic Background



- Magnetar model: protoneutron stars in very strong magnetic fields (10^{16} G) can be distorted (high ellipticity).
- Integrating over the whole universe leads to a GW background.
 - » Cutler, PRD 66, 084025 (2002).
 - » Regimbau & Mandic, CQG 25, 184018 (2008).
 - » Dall’Osso et al, MNRAS 398, 1869 (2009).
 - » Marassi et al, MNRAS 411, 2549 (2011).

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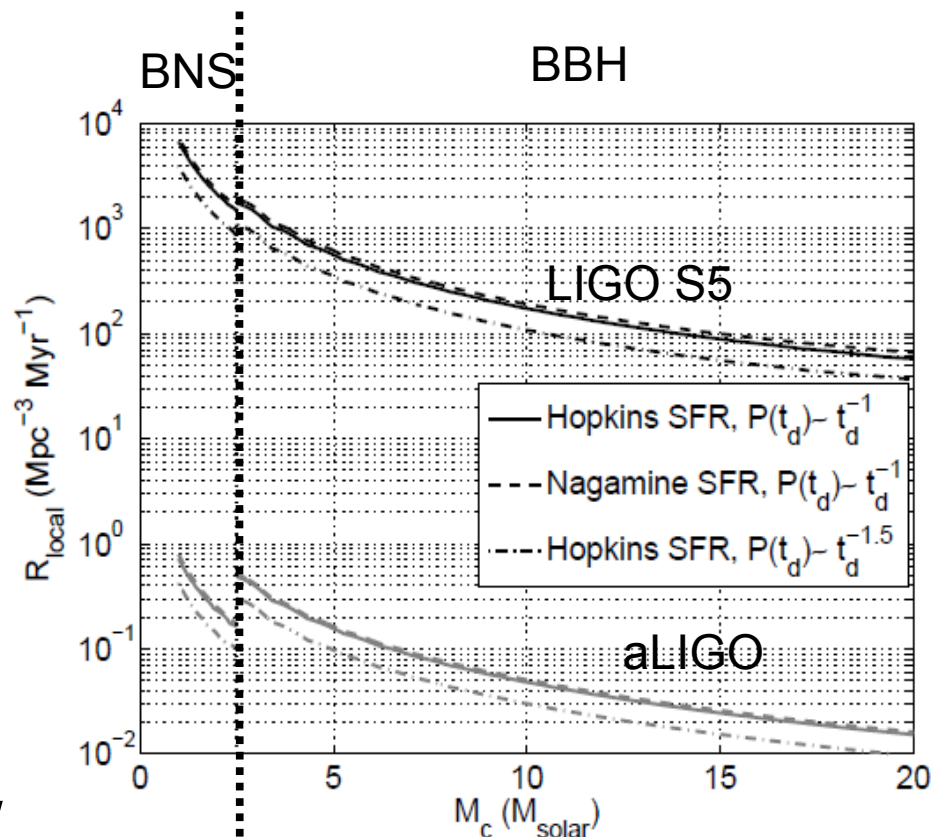


Parameter Estimation

- Perform a likelihood fit of the measured energy spectrum.

$$L(\hat{Y}_i, \sigma_i | \vec{\theta}) \propto \exp \left[-\frac{1}{2} \sum_i \frac{(\hat{Y}_i - \Omega_M(f_i; \vec{\theta}))^2}{\sigma_i^2} \right]$$

- Estimate model parameters and constrain the physics of the model.
- Example: constrain the rate of binary coalescences.
- Joint likelihood with the individual CBC observations (and others) to study the energy budget of the GW background.
- Which sources dominate? Are we missing something?



VM, E. Thrane, S. Giampanis, T. Regimbau
– in preparation

Conclusion

- GW observations are already yielding interesting astrophysical statements.
- Next-generation detectors are around the corner (2014)!
 - » Expect first direct GW observations in the coming 5 years!
- Follow-up detectors are already being planned to fully exploit the science potential of GW observations.
- Stay tuned...

